Selection and Sorting of Heterogeneous Firms Through Competitive Pressures

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Teaching Slides

1. Introduction

Competitive Pressures on Heterogeneous Firms

Main Questions: How do more ***competitive pressures***, due to entry of new firms, caused by lower ***entry cost*** or larger ***market size***, affect firms with different productivity?

- Selection of firms
- Distribution of firm size (in revenue, profit and employment), Distribution of markup and pass-through rates, etc.
- Sorting of firms across markets with different market sizes

Existing Monopolistic Competition Models with Heterogenous Firms

- Melitz (2003): under **CES** Demand System (DS)
  - MC firms sell their products at an exogenous & common markup rate, *unresponsive to competitive pressures*
  - Market size: no effect on distribution of firm types nor their behaviors; All adjustments at *the extensive margin*.
  - Firms' incentive to move across markets with different market sizes independent of firm productivity

  Inconsistent with some evidence for
  - An increase in the production cost leads to less than proportional increase in the price (the pass-through rate < 1)
  - More productive firms have higher markup rates
  - More productive firms have lower pass-through rates
- Melitz-Ottaviano (2008) departs from CES with **Linear Demand System + the outside competitive sector**, which comes with its own restrictions.

This Paper: Melitz under **H.S.A. (Homothetic Single Aggregator)** DS as a framework to study how departing from CES in the direction consistent with the evidence affects the impact of competitive pressures on heterogeneous firms.

Why H.S.A.

- **Homothetic** (unlike the linear DS and most other commonly used non-CES DSs)
  - a single measure of market size; the demand composition does not matter.
  - isolate the effect of endogenous markup rate from nonhomotheticity
  - straightforward to use it as a building block in multi-sector models with any upper-tier (incl. nonhomothetic) DS

- **Nonparametric** and **flexible** (unlike **CES** and **translog**, which are special cases)
  - can be used to perform robustness-check for CES
  - allow for (but no need to impose)
    - ✓ the choke price,
    - ✓ **Marshall's 2nd law** (Price elasticity is increasing in price) → more productive firms have higher markup rates
    - ✓ *what we call* **the 3rd law** (the rate of increase in the price elasticity is decreasing in price) → more productive firms have lower pass-through rates.

- **Tractable** due to **Single Aggregator** (unlike **Kimball**, which needs two aggregators), a *sufficient statistic* for competitive pressures, which acts like a *magnifier of firm heterogeneity*
  - guarantee the existence & uniqueness of free-entry equilibrium with firm heterogeneity
  - simple to conduct most comparative statics without *parametric* restrictions on demand or productivity distribution.
  - no need to assume zero overhead cost (unlike MO and ACDR)

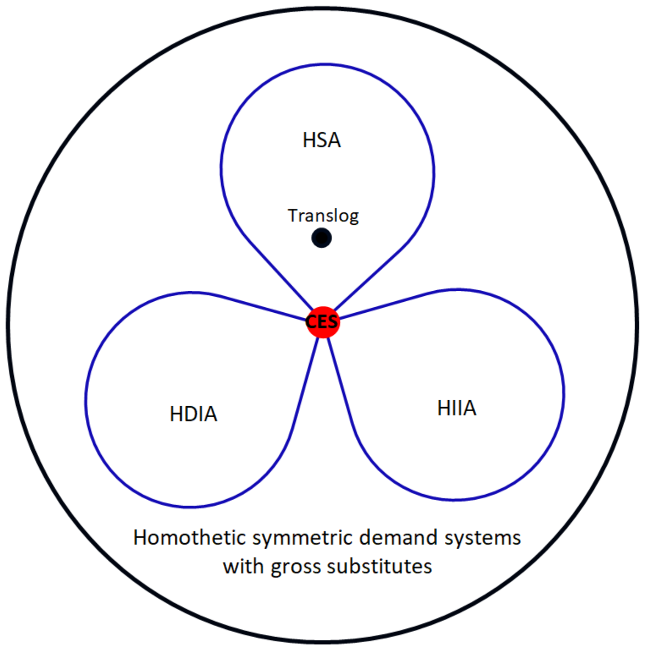
- Defined by **the market share function,** for which data is readily available and easily identifiable.

Three Classes of Homothetic Demand Systems: Matsuyama-Ushchev (2017)

Here we consider a **continuum** of varieties ($\omega \in \Omega$), **gross substitutes**, and **symmetry**

CES	$s_\omega \equiv \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} = f\left(\frac{p_\omega}{P(\mathbf{p})}\right) \Leftrightarrow s_\omega \propto \left(\frac{p_\omega}{P(\mathbf{p})}\right)^{1-\sigma}$	
H.S.A. (Homotheticity with a Single Aggregator)	$s_\omega = s\left(\frac{p_\omega}{A(\mathbf{p})}\right),$	$\frac{P(\mathbf{p})}{A(\mathbf{p})} \neq c$, unless CES
HDIA (Homotheticity with Direct Implicit Additivity) Kimball is a special case:	$s_\omega = \frac{p_\omega}{P(\mathbf{p})}(\phi')^{-1}\left(\frac{p_\omega}{B(\mathbf{p})}\right),$	$\frac{P(\mathbf{p})}{B(\mathbf{p})} \neq c$, unless CES
HIIA (Homotheticity with Indirect Implicit Additivity)	$s_\omega = \frac{p_\omega}{C(\mathbf{p})}\theta'\left(\frac{p_\omega}{P(\mathbf{p})}\right),$	$\frac{P(\mathbf{p})}{C(\mathbf{p})} \neq c$, unless CES

$\phi(\cdot)$ & $\theta(\cdot)$ are both increasing & concave → $(\phi')^{-1}(\cdot)$ & $\theta'(\cdot)$ positive-valued & decreasing.
$A(\cdot), B(\cdot), C(\cdot)$ all determined by the adding-up constraint.



The 3 classes are pairwise disjoint with the sole exception of CES.
We use HSA, because, under HDIA(Kimball) and HIIA, unlike HSA

- Two aggregators needed for the market shares. [One aggregator enough for the price elasticity under all 3 classes.]
- The existence and uniqueness of free-entry equilibrium not guaranteed without some additional restrictions on both productivity distribution and the price elasticity function.

Note: Beyond these three, "almost anything goes." E.g., Marshall's 2nd Law doesn't ensure even procompetitive entry.

Heterogeneous Firms under H.S.A.: A Summary of Main Results

- **Existence & Uniqueness of Equilibrium:** straightforward under H.S.A.

- **Under CES (i.e., Melitz)**
  - Impacts of entry/overhead costs on the masses of entrants/active firms hinges on the sign of the derivative of the elasticity of the pdf of marginal cost
  - Pareto is the knife-edge! (new results!)

- **Cross-Sectional Implications:** profits and revenues are always higher among more productive.

  - 2nd Law = incomplete pass-through ⇔ the procompetitive effect ⇔ strategic complementarity in pricing.

  - 2nd (3rd) Law → more productive firms have higher markup (lower pass-through) rates.

  - 2nd & 3rd Laws → *hump-shaped* employment; more productive hire less under high overhead.

- **Comparative Statics**
  - *Entry cost* ↓: 2nd (3rd) Law → markup rates ↓ (pass-through rates ↑) for all firms.
    profits (revenues) decline faster among less productive → a tougher selection.
  - *Overhead cost* ↓: similar effects when the employment is decreasing in firm productivity.
  - *Market size* ↑: 2nd (3rd) Law → markup rates ↓ (pass-through rates ↑) for all firms.
    profits (revenues) ↑ among more productive; ↓ among less productive.
  - *Due to the composition effect*, these changes may *increase* the average markup rate & the aggregate profit share in spite of the 2nd Law and *reduce* the average pass-through in spite of the 3rd Law; Pareto is the knife-edge *for entry cost* ↑.

- **Sorting of Heterogeneous Firms** across markets that differ in size:
  - Larger markets → more competitive pressures.
  - 2nd Law → more (less) productive go into larger (smaller) markets.
  - *Composition effect*, average markup (pass-through) rates can be *higher* (*lower*) in larger and more competitive markets in spite of 2nd (3rd) Law.

- **International/Interregional Trade with Differential Market Access**
  - 2nd Law → Exporters sell their products at lower markup rates abroad than at home..
  - Globalization (A decline in the iceberg cost):
    - share of exporting firms rise, share of domestic firms declines.
    - Exporting firms reduce their markup rate at home, increases their markup rate abroad.

(Highly Selective) Literature Review

Non-CES Demand Systems: Matsuyama (2023) for a survey; **H.S.A. Demand System:** Matsuyama-Ushchev (2017)

MC with Heterogeneous Firms: Melitz (2003) and many others: Melitz-Redding (2015) for a survey

MC under non-CES demand systems: Thisse-Ushchev (2018) , Matsuyama (2025) for a survey

- *Nonhomothetic non-CES:*
  - $U = \int_{\Omega} u(x_\omega)d\omega$: Dixit-Stiglitz (77), Behrens-Murata (07), ZKPT (12), Mrázová-Neary(17), Dhingra-Morrow (19); ACDR (19)
  - *Linear-demand system with the outside sector:* Ottaviano-Tabuchi-Thisse (2002), Melitz-Ottaviano (2008)
- *Homothetic non-CES:* Feenstra (2003), Kimball (1995), Matsuyama-Ushchev (2020a,b, 2023)
- *H.S.A.* Matsuyama-Ushchev (2022), Kasahara-Sugita (2020), Grossman-Helpman-Lhuiller (2021), Fujiwara-Matsuyama (2022), Baqaee-Fahri-Sangani (2023), Ren-Zhang (2025)

Empirical Evidence: *The 2nd Law:* DeLoecker-Goldberg (14), Burstein-Gopinath (14); *The 3rd Law:* Berman et.al.(12), Amiti et.al.(19), *Market Size Effects:* Campbell-Hopenhayn(05); *Rise of markup:* Autor et.al.(20), DeLoecker et.al.(20)

Selection of Heterogeneous Firms through Competitive Pressures
Melitz-Ottaviano (2008), Baqaee-Fahri-Sangani (2023), Edmond-Midrigan-Xu (2023)

Sorting of Heterogeneous Firms Across Markets:

- *Reduced Form/Partial Equilibrium;* Mrázová-Neary (2019), Nocke (2006)
- *General Equilibrium:* Baldwin-Okubo (2006), Behrens-Duranton-RobertNicoud (2014), Davis-Dingel (2019), Gaubert (2018), Kokovin et.al. (2022)

Log-Super(Sub)modularity: Costinot (2009), Costinot-Vogel (2015)

2. Selection of Heterogeneous Firms

2.1. The Environment: A sector producing a single final good.

Final goods producers; competitively assemble **differentiated intermediate inputs** $\omega \in \Omega$, using **CRS technology**

CRS Production Function	Unit Cost Function
$X = X(\mathbf{x}) \equiv \min_{\mathbf{p}} \left\{ \mathbf{px} \equiv \int_{\Omega} p_\omega x_\omega d\omega \,\middle|\, P(\mathbf{p}) \geq 1 \right\}$	$P = P(\mathbf{p}) \equiv \min_{\mathbf{x}} \left\{ \mathbf{px} \equiv \int_{\Omega} p_\omega x_\omega d\omega \,\middle|\, X(\mathbf{x}) \geq 1 \right\}$

Duality Theorem (or Principle):
Either $X(\mathbf{x})$ or $P(\mathbf{p})$ can be a primitive if linear homogeneity, monotonicity, quasi-concavity are satisfied.

Demand System for Differentiated Intermediate Inputs

Demand Curve (from Shepherd's Lemma)	Inverse Demand Curve
$x_\omega = \frac{\partial P(\mathbf{p})}{\partial p_\omega} X(\mathbf{x})$	$p_\omega = P(\mathbf{p}) \frac{\partial X(\mathbf{x})}{\partial x_\omega}$

$$\Longrightarrow \mathbf{px} = \int_{\Omega} p_\omega x_\omega d\omega = \int_{\Omega} \left[p_\omega \frac{\partial P(\mathbf{p})}{\partial p_\omega} \right] X(\mathbf{x}) d\omega = \int_{\Omega} P(\mathbf{p}) \left[\frac{\partial X(\mathbf{x})}{\partial x_\omega} x_\omega \right] d\omega = P(\mathbf{p}) X(\mathbf{x}) = E.$$

The total value of inputs = the total value of output under CRS = market size of this sector, E, which we treat as given.

Market Share of $\omega \in \Omega$	$s_\omega \equiv \frac{p_\omega x_\omega}{\mathbf{px}} = \frac{p_\omega x_\omega}{P(\mathbf{p}) X(\mathbf{x})} = \frac{p_\omega x_\omega}{E} = \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} = \frac{\partial \ln X(\mathbf{x})}{\partial \ln x_\omega},$

Monopolistically Competitive Intermediate Inputs Producers $\omega \in \Omega$

Essentially the same with Melitz (2003).

Each intermediate input $\omega \in \Omega$ is produced and sold exclusively by a single MC firm, also indexed by $\omega \in \Omega$.

- Sunk cost of entry, $F_e > 0$. (All costs are paid in the numeraire, "labor".)
- Each entrant draws its (quality-adjusted) marginal cost $\psi \sim G(\cdot) \in C^3$ with $G'(\psi) = g(\psi) > 0$ on $\left(\underline{\psi}, \overline{\psi}\right) \subseteq (0, \infty)$.
  $\mathcal{E}_G(\psi) \equiv \psi g(\psi)/G(\psi) \in C^2$ and $\mathcal{E}_g(\psi) \equiv \psi g'(\psi)/g(\psi) \in C^1$.

MC firms are ex-ante homogeneous but become ex-post heterogeneous *only* in ψ, or equivalently, in (quality-adjusted) productivity, $1/\psi = \varphi \sim 1 - G(1/\varphi)$ with density $g(1/\varphi)/\varphi^2 > 0$ on $\left(\underline{\varphi}, \overline{\varphi}\right) \subseteq (0, \infty)$.

- Upon discovering its marginal cost, , ψ_ω, firm ω calculates its gross profit, $\Pi(\psi_\omega)$, after learning its marginal cost.

- Firms that stay will have to pay an overhead cost, $F > 0$.
  - If $\Pi(\psi_\omega) \geq F$, it chooses to stay, and earns net profit, $\Pi(\psi_\omega) - F$.
  - If $\Pi(\psi_\omega) > F$, it chooses to exit without paying $F > 0$, and earns zero net profit.

- Free entry by (ex-ante homogeneous) firms: $\int \max\{\Pi(\psi) - F, 0\}\, dG(\psi) = F_e$.

This ensures that the total demand for the numeraire is equal to market size, $L = E$.

2.2. Symmetric H.S.A. (Homothetic with a Single Aggregator) with Gross Substitutes

Market Share of ω depends *solely* on a single variable, its own price normalized by the *common* price aggregator

$$s_\omega \equiv \frac{p_\omega x_\omega}{P(\mathbf{p})X(\mathbf{x})} = \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} = s\left(\frac{p_\omega}{A(\mathbf{p})}\right), \qquad \text{where} \qquad \int_\Omega s\left(\frac{p_\omega}{A(\mathbf{p})}\right) d\omega \equiv 1.$$

- $s: \mathbb{R}_{++} \to \mathbb{R}_+$: **the market share function**, C^3, decreasing in the **normalized price;** $z_\omega \equiv p_\omega / A$ for $s(z_\omega) > 0$ with
  - $\lim_{z \to \bar{z}} s(z) = 0$. If $\bar{z} \equiv \inf\{z > 0 | s(z) = 0\} < \infty$, $\bar{z}A(\mathbf{p})$ is the **choke price.**

- $A = A(\mathbf{p})$: the **common price aggregator** defined implicitly by **the adding up constraint** $\int_\Omega s(p_\omega / A) d\omega \equiv 1$.
  By construction, $A(\mathbf{p})$ has to be linear homogenous in $\mathbf{p}$ for a fixed Ω. A larger Ω reduces $A(\mathbf{p})$.

Special Cases	CES	$s(z) = \gamma z^{1-\sigma}$;	$\sigma > 1$
	Translog	$s(z) = -\gamma \max\left\{\ln\left(\frac{z}{\bar{z}}\right), 0\right\}$;	$\bar{z} < \infty$
	Constant Pass Through (CoPaTh)	$s(z) = \gamma \max\left\{\left[\sigma + (1-\sigma) z^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}, 0\right\}$	$0 < \rho < 1$

As $\rho \nearrow 1$, CoPaTh converges to CES with $\bar{z}(\rho) \equiv (\sigma/(\sigma-1))^{\frac{\rho}{1-\rho}} \to \infty$.

$P(\mathbf{p})$ vs. $A(\mathbf{p})$

Definition: $s_\omega \equiv \dfrac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} = s\left(\dfrac{p_\omega}{A(\mathbf{p})}\right) \equiv s(z_\omega),$ where $\int_\Omega s\left(\dfrac{p_\omega}{A(\mathbf{p})}\right) d\omega \equiv \int_\Omega s(z_\omega) d\omega \equiv 1.$

By differentiating the adding-up constraint, $\dfrac{\partial \ln A(\mathbf{p})}{\partial \ln p_\omega} = \dfrac{[\zeta(z_\omega)-1]s(z_\omega)}{\int_\Omega [\zeta(z_{\omega'})-1]s(z_{\omega'})d\omega'} \neq s(z_\omega) = \dfrac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega}$

unless $\zeta(z_\omega)$ is constant, where

Price Elasticity Function: $\zeta(z) \equiv 1 - \dfrac{d \ln s(z)}{d \ln z} \equiv 1 - \mathcal{E}_s(z) > 1 \Leftrightarrow s(z) = \gamma \exp\left[\int_{z_0}^{z} \dfrac{1-\zeta(\xi)}{\xi} d\xi\right]$ for $z \in (0, \bar{z})$; $\lim_{z\to\bar{z}} \zeta(z) = \infty$, if $\bar{z} < \infty$.

By integrating the definition, $\dfrac{cP(\mathbf{p})}{A(\mathbf{p})} = \exp\left[-\int_\Omega s\left(\dfrac{p_\omega}{A(\mathbf{p})}\right)\Phi\left(\dfrac{p_\omega}{A(\mathbf{p})}\right) d\omega\right],$ where $\Phi(z) \equiv \dfrac{1}{s(z)}\int_z^{\bar{z}} \dfrac{s(\xi)}{\xi} d\xi > 0.$

$c > 0$: The integral constant, proportionally to TFP.
$\Phi(z) > 0$: Productivity gain from a product sold at $z > 0$, satisfying $\zeta'(\cdot) \gtreqless 0 \implies \Phi'(\cdot) \lesseqgtr 0$; $\Phi'(\cdot) = 0 \Leftrightarrow \zeta'(\cdot) = 0$.
$P(\mathbf{p})$ satisfies linear homogeneity, monotonicity, and quasi-concavity, and symmetry.

Note: $P(\mathbf{p})/A(\mathbf{p})$ is not constant, unless CES $\Leftrightarrow \zeta(z) = \sigma \Leftrightarrow s(z) = \gamma z^{1-\sigma} \Leftrightarrow \Phi(z) = 1/(\sigma - 1)$.

- ✓ $A(\mathbf{p})$, the inverse measure of *competitive pressures*, captures *cross price effects* in the demand system.
- ✓ $P(\mathbf{p})$, the inverse measure of TFP, captures the *productivity consequences* of price changes.

2.3. MC firms under H.S.A.: Each firm takes $A = A(\mathbf{p})$ and E given.

$$\Pi_\omega \equiv \max_{p_\omega}(p_\omega - \psi_\omega)x_\omega = \max_{\psi_\omega < p_\omega < \bar{z}A}\left(1 - \frac{\psi_\omega}{p_\omega}\right) s\left(\frac{p_\omega}{A}\right) E = \max_{\psi_\omega/A < z_\omega < \bar{z}}\left(1 - \frac{\psi_\omega/A}{z_\omega}\right) s(z_\omega)E.$$

FOC:

$$z_\omega\left[1 - \frac{1}{\zeta(z_\omega)}\right] = \frac{\psi_\omega}{A}$$

We maintain the following assumption *for ease of exposition*.

(A1): For all $z \in (0, \bar{z})$,

$$\mathcal{E}_{z(\zeta-1)/\zeta}(z) > 0 \Leftrightarrow \mathcal{E}_{\zeta/(\zeta-1)}(z) < 1 \Leftrightarrow \mathcal{E}_{s/\zeta}(z) = \mathcal{E}_s(z) - \mathcal{E}_\zeta(z) < 0$$

- **(A1)** holds if $\zeta(\cdot)$ is increasing. i.e., under **Marshall's 2nd Law, (A2)**
- **(A1)** means that LHS of FOC, the marginal revenue, is strictly increasing in p_ω (hence strictly decreasing in x_ω)
  → FOC determines the profit maximizing z_ω as an increasing C^2 function of ψ_ω/A.

Without (A1), it is still increasing, but only piecewise-C^2 (i.e., the price would jump up at some values of ψ)

  → Firms with the same ψ set the same price, earn the same profit → we index firms by ψ, as p_ψ, $z_\psi \equiv p_\psi/A$.

- **(A1)** ensures that the maximized profit Π_ω is a decreasing C^2 function of ψ_ω/A.

Without (A1), the maximized profit is decreasing, but only piecewise-C^1.

Markup and Pass-Through Rates

Lerner Pricing Formula:
$$z_\psi \left[1 - \frac{1}{\zeta(z_\psi)}\right] = \frac{\psi}{A}$$

Under **A1**, LHS is strictly increasing, so the Inverse Function Theorem allows us to rewrite it as

Normalized Price:
$$\frac{p_\psi}{A} \equiv z_\psi = Z\left(\frac{\psi}{A}\right) \in (\psi/A\,, \bar{z}); \qquad Z'(\cdot) > 0;$$

Price Elasticity: $\zeta(z_\psi) = \zeta\left(Z\left(\frac{\psi}{A}\right)\right) \equiv \sigma\left(\frac{\psi}{A}\right) > 1;$ **Markup Rate:** $\mu_\psi \equiv \frac{p_\psi}{\psi} = \frac{\sigma(\psi/A)}{\sigma(\psi/A) - 1} \equiv \mu\left(\frac{\psi}{A}\right) > 1$

satisfying
$$\frac{1}{\sigma(\psi/A)} + \frac{1}{\mu(\psi/A)} = 1 \Leftrightarrow \left[\sigma\left(\frac{\psi}{A}\right) - 1\right]\left[\mu\left(\frac{\psi}{A}\right) - 1\right] = 1$$

Pass-Through Rate:
$$\rho_\psi \equiv \frac{\partial \ln p_\psi}{\partial \ln \psi} = \mathcal{E}_Z\left(\frac{\psi}{A}\right) \equiv \rho\left(\frac{\psi}{A}\right) = 1 + \mathcal{E}_\mu\left(\frac{\psi}{A}\right) = 1 - \frac{\mathcal{E}_\sigma(\psi/A)}{\sigma(\psi/A) - 1} > 0$$

- Normalized price, and markup rate, all C^2 functions of the *normalized cost,* ψ/A only.
    - $Z'(\cdot) > 0$; always strictly increasing in ψ/A; Markup rate, can be increasing, decreasing or nonmonotone.
- Pass-through rate, a C^1 function of ψ/A only, can be increasing, decreasing, or nonmonotone in general.
- Market size affects the pricing behaviors of firms only through its effects on A.
- More competitive pressures, a lower A, act like a magnifier of firm heterogeneity.

Under CES, $\sigma(\cdot) = \sigma;\ \mu(\cdot) = \sigma/(\sigma - 1) = \mu;\ \rho(\cdot) = 1.$

Revenue, Profit, and Employment

Revenue	(Gross) Profit	(Variable) Employment
$R_\psi = s(z_\psi)E = s\left(\tilde{Z}\left(\frac{\psi}{A}\right)\right)E \equiv r\left(\frac{\psi}{A}\right)E$	$\Pi_\psi = \frac{r(\psi/A)}{\sigma(\psi/A)}E \equiv \pi\left(\frac{\psi}{A}\right)E$	$\psi x_\psi = \frac{r(\psi/A)}{\mu(\psi/A)}E \equiv \ell\left(\frac{\psi}{A}\right)E$
$\frac{\partial \ln R_\psi}{\partial \ln(\psi/A)} \equiv \mathcal{E}_r\left(\frac{\psi}{A}\right) = \left[1-\sigma\left(\frac{\psi}{A}\right)\right]\rho\left(\frac{\psi}{A}\right) < 0$ Always strictly negative.	$\frac{\partial \ln \Pi_\psi}{\partial \ln(\psi/A)} \equiv \mathcal{E}_\pi\left(\frac{\psi}{A}\right) = 1-\sigma\left(\frac{\psi}{A}\right) < 0$ Always strictly negative.	$\frac{\partial \ln(\psi x_\psi)}{\partial \ln(\psi/A)} \equiv \mathcal{E}_\ell\left(\frac{\psi}{A}\right) = 1-\sigma\left(\frac{\psi}{A}\right)\rho\left(\frac{\psi}{A}\right)$ Nonmonotone in general.
$\frac{\partial^2 \ln R_\psi}{\partial\psi\partial(1/A)} = \left[1-\sigma\left(\frac{\psi}{A}\right)\right]\rho'\left(\frac{\psi}{A}\right) - \sigma'\left(\frac{\psi}{A}\right)\rho\left(\frac{\psi}{A}\right)$ Negative under the 2nd & weak 3rd laws	$\frac{\partial^2 \ln \Pi_\psi}{\partial\psi\partial(1/A)} = -\sigma'\left(\frac{\psi}{A}\right)$ Negative under the 2nd law	$\frac{\partial^2 \ln(\psi x_\psi)}{\partial\psi\partial(1/A)} = -\sigma'\left(\frac{\psi}{A}\right)\rho\left(\frac{\psi}{A}\right) - \sigma\left(\frac{\psi}{A}\right)\rho'\left(\frac{\psi}{A}\right)$ Negative under the 2nd & the weak 3rd laws

- Revenue, profit, employment are all C^2 functions of ψ/A, multiplied by **market size** E.
- $\mathcal{E}_r(\cdot), \mathcal{E}_\pi(\cdot)$ and $\mathcal{E}_\ell(\cdot)$ depend solely on $\sigma(\cdot)$ and $\rho(\cdot)$, hence all C^1 functions of ψ/A only.

  More competitive pressures, a lower A, act like a magnifier of firm heterogeneity.

  Market size affects the relative profit, revenue, and employment across firms only through its effects on A.

Under CES, $r(\cdot)/\pi(\cdot) = \sigma$; $r(\cdot)/\ell(\cdot) = \mu = \sigma/(\sigma-1) \Longrightarrow \mathcal{E}_r(\cdot) = \mathcal{E}_\pi(\cdot) = \mathcal{E}_\ell(\cdot) = 1-\sigma < 0$.

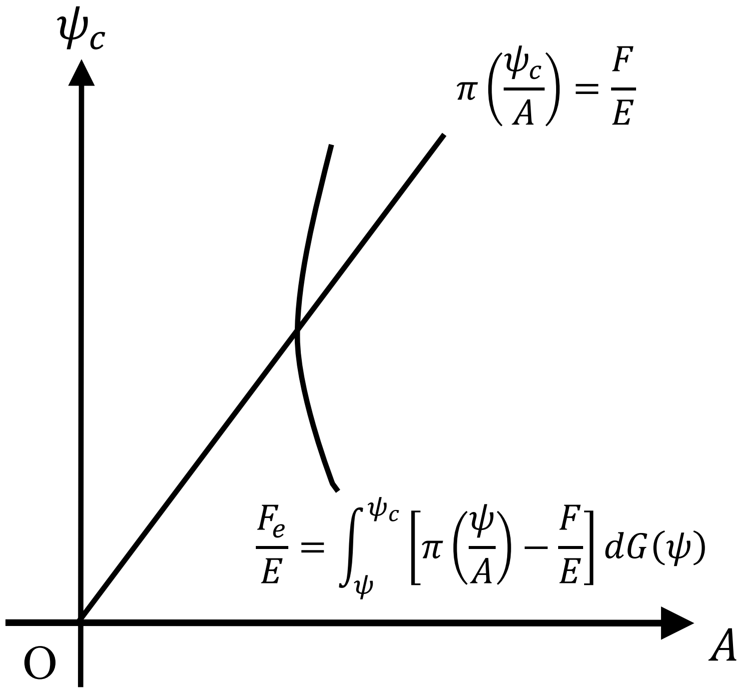
- Both revenue and profit are always strictly decreasing in ψ/A.
- Employment $\ell(\psi/A)E$ may be nonmonotonic in ψ/A.

2.4 Equilibrium Condition: Assume $F + F_e < \pi(0)E$.

Cutoff Rule: Stay if $\psi < \psi_c$; exit if $\psi > \psi_c$, where

$$\max_{\psi_c} \int_{\underline{\psi}}^{\psi_c} \left[\pi\left(\frac{\psi}{A}\right)E - F\right] dG(\psi) \Longrightarrow \pi\left(\frac{\psi_c}{A}\right)E = F$$

positively-sloped. $A \downarrow$ (more competitive pressures) $\Longrightarrow \psi_c \downarrow$ (tougher selection)
rotate clockwise, as $F/E \uparrow$ (higher overhead/market size) $\Longrightarrow \psi_c/A \downarrow$.



Free Entry Condition:

$$F_e = \int_{\underline{\psi}}^{\psi_c} \left[\pi\left(\frac{\psi}{A}\right)E - F\right] dG(\psi)$$

shift to the left as $F_e \downarrow$ (lower entry cost) $\Longrightarrow A \downarrow$ (more competitive pressures).

$A = A(\mathbf{p})$ and ψ_c: uniquely determined as C^2 functions of F_e/L & F/L with the interior solution, $0 < G(\psi_c) < 1$ for

$$0 < \frac{F_e}{E} < \int_{\underline{\psi}}^{\overline{\psi}} \left[\pi\left(\pi^{-1}\left(\frac{F}{E}\right)\frac{\psi}{\overline{\psi}}\right) - \frac{F}{E}\right] dG(\psi),$$

which holds for a sufficiently small $F_e > 0$ with no further restrictions on $G(\cdot)$ and $s(\cdot)$.
(This unique existence proof applies also to the Melitz model, which assumes CES.)

From the adding-up (resource) constraint, $1 \equiv \int_{\Omega} s\left(\frac{p_\omega}{A}\right) d\omega = \int_{\underline{\psi}}^{\psi_c} r\left(\frac{\psi}{A}\right) M dG(\psi)$,

Mass of Active Firms
= the measure of Ω

$$MG(\psi_c) = \left[\int_{\underline{\psi}}^{\psi_c} r\left(\frac{\psi}{A}\right)\frac{dG(\psi)}{G(\psi_c)}\right]^{-1} = \left[\int_{\underline{\xi}}^{1} r\left(\pi^{-1}\left(\frac{F}{E}\right)\xi\right) d\tilde{G}(\xi;\psi_c)\right]^{-1} > 0,$$

where $\tilde{G}(\xi;\psi_c) \equiv \frac{G(\psi_c\xi)}{G(\psi_c)}$ is the cdf of $\xi \equiv \psi/\psi_c$, conditional on $\underline{\xi} \equiv \underline{\psi}/\psi_c < \xi \leq 1$.

Lemma 1: $\mathcal{E}_g'(\psi) < 0 \Longrightarrow \mathcal{E}_G'(\psi) < 0$; $\mathcal{E}_g'(\psi) \geq 0 \Longrightarrow \mathcal{E}_G'(\psi) \geq 0$ with some boundary conditions.
Lemma 2: A lower ψ_c shifts $\tilde{G}(\xi;\psi_c)$ to the right (left) in MLR if $\mathcal{E}_g'(\psi) < (>)0$ and in FSD if $\mathcal{E}_G'(\psi) < (>)0$.

- Some evidence for $\mathcal{E}_g'(\psi) > 0 \Longrightarrow \psi_c \downarrow$ (tougher selection) shifts $\tilde{G}(\xi;\psi_c)$ to the left.
- Pareto-productivity, $G(\psi) = (\psi/\bar{\psi})^\kappa \Longrightarrow \mathcal{E}_g'(\psi) = \mathcal{E}_G'(\psi) = 0 \Longrightarrow \tilde{G}(\xi;\psi_c)$ is independent of ψ_c.
- Fréchet, Weibull, Lognormal; $\mathcal{E}_g'(\psi) < 0 \Longrightarrow \mathcal{E}_G'(\psi) < 0 \Longrightarrow \psi_c \downarrow$ (tougher selection) shifts $\tilde{G}(\xi;\psi_c)$ to the right.

Lemma 4: The integrals in the equilibrium conditions are finite and hence the equilibrium is well-defined, if $\underline{\psi} > 0 \Leftrightarrow \overline{\varphi} < \infty$ or $1 + \lim_{z\to 0}\zeta(z) < 2 + \lim_{\psi\to 0}\mathcal{E}_g(\psi) = -\lim_{\varphi\to\infty}\mathcal{E}_f(\varphi) < \infty$ for $\underline{\psi} = 0 \Leftrightarrow \overline{\varphi} = \infty$.

Notes:

- **Equilibrium can be solved recursively under H.S.A.** Under HDIA/HIIA, one needs to solve the 3 equations simultaneously for 3 variables, ψ_c & the two price aggregates.
- **A sector-wide productivity shock, $G(\psi) \to G(\psi/\lambda)$**: causes $\psi_c \to \lambda\psi_c$, $A \to \lambda A$, leaving ψ_c/A, hence, the markup and the pass-through rates, the profit, the revenue, and the employment distributions across firms unchanged

2.5 Aggregate Labor Cost and Profit Shares and TFP

Notations:

The $w(\cdot)$-weighted average of $f(\cdot)$ among the active firms, $\psi \in (\underline{\psi}, \psi_c)$	$\mathbb{E}_w(f) \equiv \dfrac{\int_{\underline{\psi}}^{\psi_c} f\left(\frac{\psi}{A}\right) w\left(\frac{\psi}{A}\right) dG(\psi)}{\int_{\underline{\psi}}^{\psi_c} w\left(\frac{\psi}{A}\right) dG(\psi)}.$
The unweighted average of $f(\cdot)$ among the active firms, $\psi \in (\underline{\psi}, \psi_c)$	$\mathbb{E}_1(f) \equiv \dfrac{\int_{\underline{\psi}}^{\psi_c} f\left(\frac{\psi}{A}\right) dG(\psi)}{\int_{\underline{\psi}}^{\psi_c} dG(\psi)}.$

$$\Longrightarrow \mathbb{E}_w\left(\frac{f}{w}\right) = \frac{\mathbb{E}_1(f)}{\mathbb{E}_1(w)} = \left[\mathbb{E}_f\left(\frac{w}{f}\right)\right]^{-1}.$$

By applying the above formulae to $\pi(\cdot)/r(\cdot) = 1 - \ell(\cdot)/r(\cdot) = 1/\sigma(\cdot) = 1 - 1/\mu(\cdot)$,

Aggregate Labor Cost Share (Average inverse markup rate)	$\dfrac{\mathbb{E}_1(\ell)}{\mathbb{E}_1(r)} = \mathbb{E}_r\left(\dfrac{1}{\mu}\right) = 1 - \left[\mathbb{E}_\pi\left(\dfrac{\mu}{\mu-1}\right)\right]^{-1} = \dfrac{1}{\mathbb{E}_\ell(\mu)}$
Aggregate Profit Share (Average inverse price elasticity)	$\dfrac{\mathbb{E}_1(\pi)}{\mathbb{E}_1(r)} = \mathbb{E}_r\left(\dfrac{1}{\sigma}\right) = \dfrac{1}{\mathbb{E}_\pi(\sigma)} = 1 - \left[\mathbb{E}_\ell\left(\dfrac{\sigma}{\sigma-1}\right)\right]^{-1}$
Aggregate TFP	$\ln\left(\dfrac{X}{L}\right) = \ln\left(\dfrac{1}{P}\right) = \ln\left(\dfrac{c}{A}\right) + \mathbb{E}_r[\Phi \circ Z]$

3. CES Benchmark: Revisiting Melitz

CES Benchmark: For all $z \in (0, \infty)$, $\zeta(z) = \sigma > 1 \Leftrightarrow s(z) = \gamma z^{1-\sigma}$.

Pricing: $p_\psi\left(1 - \frac{1}{\sigma}\right) = \psi \Leftrightarrow \mu\left(\frac{\psi}{A}\right) = \frac{\sigma}{\sigma - 1} > 1 \Rightarrow \rho\left(\frac{\psi}{A}\right) = 1$

Markup rate constant; Pass-through rate equal to one.

Cutoff Rule: $$c_0 E\left(\frac{\psi_c}{A}\right)^{1-\sigma} = F,$$

Free Entry Condition: $$\int_{\underline{\psi}}^{\psi_c}\left[c_0 E\left(\frac{\psi}{A}\right)^{1-\sigma} - F\right] dG(\psi) = F_e,$$

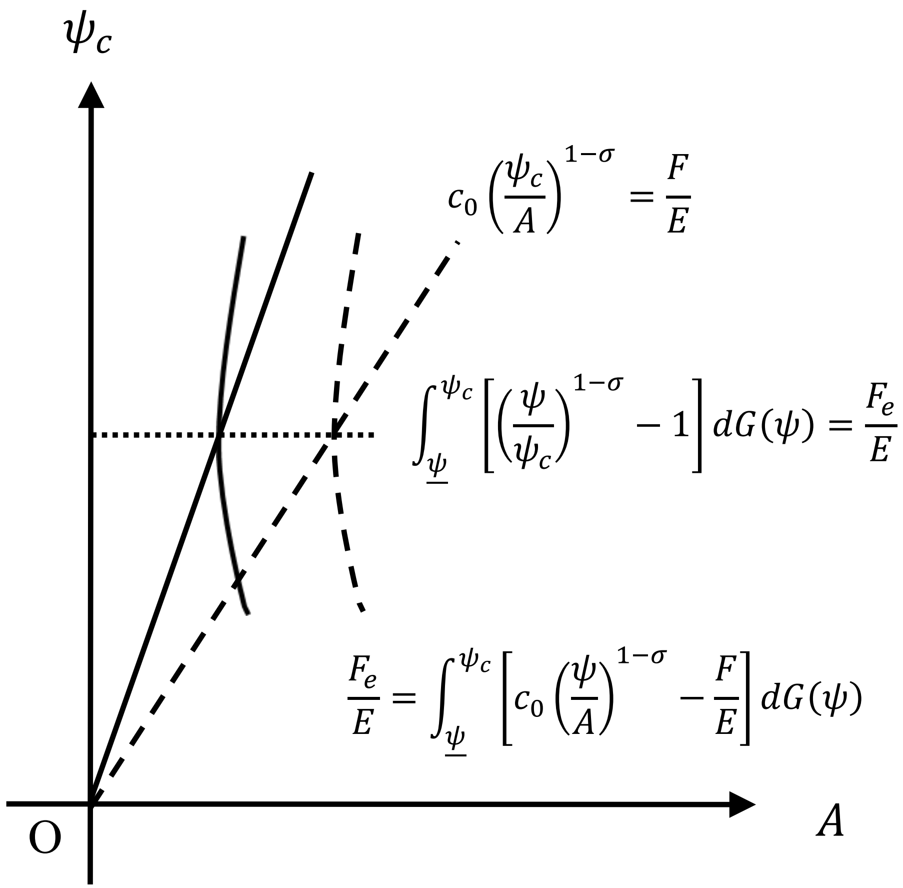
with $c_0 > 0$. As E changes, the intersection moves along

$$\int_{\underline{\psi}}^{\psi_c}\left[\left(\frac{\psi}{\psi_c}\right)^{1-\sigma} - 1\right] dG(\psi) = \frac{F_e}{F}$$

$F_e/F \downarrow$ and a FSD shift of $G(\cdot)$ *to the left* $\Longrightarrow \psi_c \downarrow$ (tougher selection).

ψ_c unaffected by E, and independent of A.

$$A = \psi_c\left(\frac{c_0 E}{F}\right)^{\frac{1}{1-\sigma}} = \left(\frac{c_0 E}{F_e}\int_{\underline{\psi}}^{\psi_c}[(\psi)^{1-\sigma} - (\psi_c)^{1-\sigma}]dG(\psi)\right)^{\frac{1}{1-\sigma}}.$$



$E \uparrow$, $F_e \downarrow$, $F \downarrow$, a FSD shift of $G(\cdot)$ to the left $\Longrightarrow A \downarrow$ (more competitive pressures)

CES Benchmark (Continue)

Revenue:
$$r\left(\frac{\psi}{A}\right)E = \sigma c_0 E\left(\frac{\psi}{A}\right)^{1-\sigma} = \sigma F\left(\frac{\psi}{\psi_c}\right)^{1-\sigma} \geq \sigma F$$

(Gross) Profi:
$$\pi\left(\frac{\psi}{A}\right)E = c_0 E\left(\frac{\psi}{A}\right)^{1-\sigma} = F\left(\frac{\psi}{\psi_c}\right)^{1-\sigma} \geq F$$

(Variable) Employment:
$$\ell\left(\frac{\psi}{A}\right)E = (\sigma-1)c_0 E\left(\frac{\psi}{A}\right)^{1-\sigma} = (\sigma-1)F\left(\frac{\psi}{\psi_c}\right)^{1-\sigma} \geq (\sigma-1)F$$

All decreasing **power** functions of ψ with

$$\mathcal{E}_r\left(\frac{\psi}{A}\right) = \mathcal{E}_\pi\left(\frac{\psi}{A}\right) = \mathcal{E}_\ell\left(\frac{\psi}{A}\right) = 1-\sigma < 0.$$

Relative size of two firms with $\psi,\ \psi' \in (\underline{\psi}, \psi_c)$, whether measured in the profit, employment, and revenue, unaffected by E, F_e, F, $G(\cdot)$, as well as A and ψ_c, and thus never change across equilibriums.

CES Benchmark (Continue)

Mass of entrants $$M = \frac{E/\sigma}{F_e + G(\psi_c)F} = \frac{E}{\sigma F_e}\left[1 - \frac{1}{H(\psi_c)}\right]$$

Mass of active firms $$MG(\psi_c) = \frac{E/\sigma}{F_e/G(\psi_c) + F} = \frac{E}{H(\psi_c)\sigma F}$$

where $H(\psi_c) \equiv \int_{\underline{\xi}}^{1}(\xi)^{1-\sigma}\tilde{G}(\xi;\psi_c)$. Since $(\xi)^{1-\sigma}$ is decreasing, $\mathcal{E}'_G(\psi) \lesseqgtr 0 \Longrightarrow H'(\psi_c) \gtreqless 0$ (Lemma 2).

Hence,

Proposition 1: Under CES,
- $E \uparrow$ keeps ψ_c unaffected; increases both M and $MG(\psi_c)$ *proportionately;*
- $F_e \downarrow$ reduces ψ_c; increases M; increases (decreases) $MG(\psi_c)$ if $\mathcal{E}'_G(\psi) < (>)0$;
- $F \downarrow$ increases ψ_c; increases $MG(\psi_c)$; increases (decreases) M if $\mathcal{E}'_G(\psi) < (>)0$;

A FSD shift of $G(\cdot)$ to the left reduces ψ_c with ambiguous effects on M and $MG(\psi_c)$, even if $G(\cdot)$ is a power.

Effects of Market Size *E* under CES:
- No effect on the markup rate.
- No effect on the cutoff, ψ_c
- No effect on the distribution of productivity, revenue, and employment across firms.
- Masses of entrants and of active firms change ***proportionately.*** All adjustments at ***the extensive margin.***

4. Heterogeneous Firms under H.S.A.: Cross-Sectional Implications

4.1 Cross Sectional Implications of the 2^{nd} Law (A2)

(A2): $\zeta'(z) > 0$ for all $z \in (0, \bar{z}) \Leftrightarrow \sigma'(\psi/A) > 0$ for all $\psi/A \in (0, \bar{z})$

Note: **A2 ⇒ A1.**

Lemma 5: For a positive-valued function of a single variable, $\psi/A > 0$,

$$sgn\left\{\frac{\partial^2 \ln f(\psi/A)}{\partial\psi\partial(1/A)}\right\} = sgn\left\{\mathcal{E}_f'\left(\frac{\psi}{A}\right)\right\} = sgn\left\{\frac{d^2 \ln f\left(e^{\ln(\psi/A)}\right)}{(d\ln(\psi/A))^2}\right\}$$

$f(\psi/A)$ *log-super(sub)modular* in ψ & $1/A \Leftrightarrow \mathcal{E}_f'(\cdot) > (<)0 \Leftrightarrow \ln f\left(e^{\ln(\psi/A)}\right)$ convex (concave) in $\ln(\psi/A)$.

Proposition 2: Under **A2,**

Incomplete Pass-Through

$$0 < \frac{\partial \ln p_\psi}{\partial \ln \psi} = \rho\left(\frac{\psi}{A}\right) = 1 + \mathcal{E}_\mu\left(\frac{\psi}{A}\right) = 1 - \mathcal{E}_{1/\mu}\left(\frac{\psi}{A}\right) < 1$$

Less efficient firms operate at more elastic parts of demand and have lower markup rates

Procompetitive Effect/
Strategic Complementarity in Pricing

$$\frac{\partial \ln p_\psi}{\partial \ln(1/A)} = \rho\left(\frac{\psi}{A}\right) - 1 = \mathcal{E}_\mu\left(\frac{\psi}{A}\right) = -\mathcal{E}_{1/\mu}\left(\frac{\psi}{A}\right) < 0$$

More competitive pressures ($A \downarrow$ due to entry or lower prices of competing products) → lower prices/markup rates.

Strict Log-submodular Profit:

$$\mathcal{E}_\pi'\left(\frac{\psi}{A}\right) < 0 \Leftrightarrow \frac{\partial^2 \ln \pi(\psi/A)E}{\partial\psi\partial(1/A)} < 0$$

More competitive pressures ($A \downarrow$) → a proportionately larger decline in the profit among high-ψ firms
→ a larger dispersion of the profit across firms; more concentration of profits among the productive.

4.2 Cross-Sectional Implications of the 3rd Law (A3)

(A3) **(A3)**: **Weak** **(Strong)** **3rd Law of demand**. For all $z \in (0, \bar{z})$,

$$\mathcal{E}'_{\zeta/(\zeta-1)}(z) = -\frac{d}{dz}\left(\frac{z\zeta'(z)}{[\zeta(z)-1]\zeta(z)}\right) \geq (>) 0 \Leftrightarrow \rho'\left(\frac{\psi}{A}\right) = \mathcal{E}'_Z\left(\frac{\psi}{A}\right) = \mathcal{E}'_\mu\left(\frac{\psi}{A}\right) \geq (>) 0$$

Strong A3 → The markup rate declines at a lower rate for higher z → The pass-through rate higher for higher ψ.

- **A3** has some empirical support. Translog violates **A3**. CoPaTh satisfies **A3** but not **A3.** PEM satisfies **A3.**

Proposition 3: Under **A3(A3),**

Weak (Strict) log-supermodular markup rate:

$$\mathcal{E}'_Z\left(\frac{\psi}{A}\right) = \rho'\left(\frac{\psi}{A}\right) \geq (>) < 0 \Leftrightarrow \frac{\partial^2 \ln\left(Z(\psi/A)\right)}{\partial\psi\partial(1/A)} = \frac{\partial^2 \ln \mu(\psi/A)}{\partial\psi\partial(1/A)} \geq (>) 0,$$

For the strict 3rd law, more competitive pressures ($A \downarrow$) → proportionately smaller rate decline among high-ψ firms.
→ a smaller dispersion of the markup rate across firms.

Under **A2**+**A3**

Strict Log-submodular Revenue:

$$\mathcal{E}'_r\left(\frac{\psi}{A}\right) = \left[1 - \sigma\left(\frac{\psi}{A}\right)\right]\rho'\left(\frac{\psi}{A}\right) - \sigma'\left(\frac{\psi}{A}\right)\rho\left(\frac{\psi}{A}\right) < 0 \Leftrightarrow \frac{\partial^2 \ln r(\psi/A)E}{\partial\psi\partial(1/A)} < 0$$

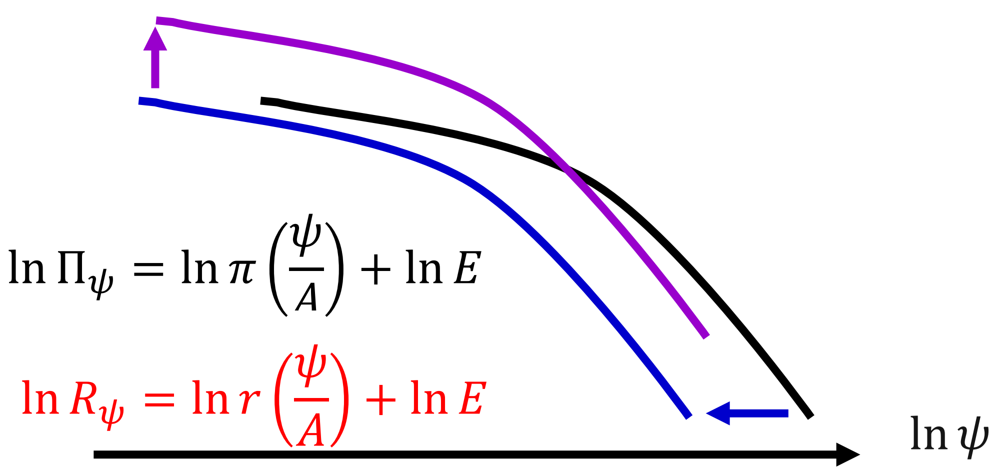
Strict Log-submodular employment:

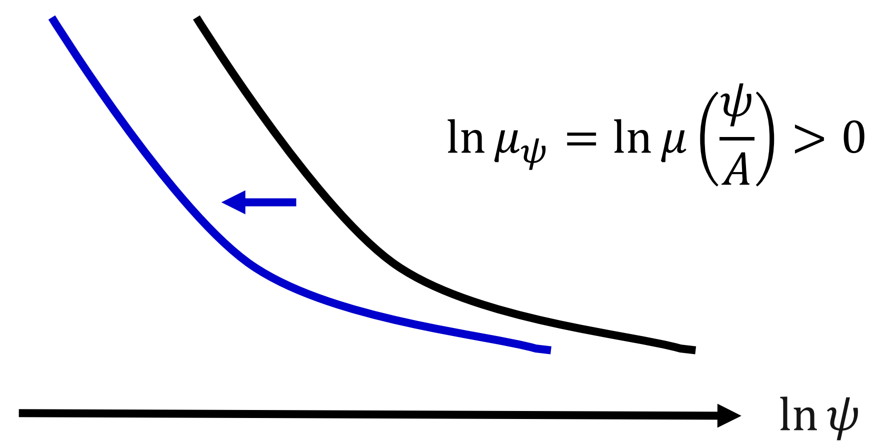
$$\mathcal{E}'_\ell\left(\frac{\psi}{A}\right) = -\sigma\left(\frac{\psi}{A}\right)\rho'\left(\frac{\psi}{A}\right) - \sigma'\left(\frac{\psi}{A}\right)\rho\left(\frac{\psi}{A}\right) < 0 \Leftrightarrow \frac{\partial^2 \ln \ell(\psi/A)E}{\partial\psi\partial(1/A)} < 0.$$

More competitive pressures ($A \downarrow$) → proportionately larger decline in the revenue among high-ψ firms
→ a larger dispersion of the revenue across firms; more concentration of revenue among the productive.

A2+A3: Cross-Sectional Implications of $A \downarrow$ on Profit and Markup Rate

Profit (Revenue) Function: $\Pi_\psi = \pi(\psi/A)E$; $R_\psi = r(\psi/A)E$	Markup Rate Function: $\mu_\psi = \mu(\psi/A) > 1$
• *always* decreasing in ψ • strictly log-submodular *under A2 (Weak A3)* → $A \downarrow$ with E fixed shifts down with a steeper slope at each ψ; → $A \downarrow$ due to $E \uparrow$, a parallel shift up, a *single-crossing*	• decreasing in ψ *under A2* • weakly log-supermodular *under Weak A3* • strictly log-supermodular *under Strong A3* → $A \downarrow$ shifts down with a flatter slope at each ψ





✓ With $\ln \psi$ in the horizontal axis, $A \downarrow$ causes a parallel leftward shift of the graphs in these figures.
✓ $f(\psi/A)$ is strictly log-super(sub)modular in ψ and $1/A$ iff $\ln f(e^x)$ is convex(concave) in x.

A2+A3: More Cross-Sectional Implications

Lemma 6: Under A2 and the weak A3, $\lim_{\psi/A\to 0}\rho(\psi/A)\sigma(\psi/A) < 1 < \lim_{\psi/A\to\bar{z}}\rho(\psi/A)\sigma(\psi/A)$.

Since **A2+A3** also implies $\mathcal{E}'_\ell(\psi/A) < 0$,

Proposition 4: Under **A2** and the wea**k A3,** the employment function, $\ell(\psi/A) = r(\psi/A)/\mu(\psi/A)$ is hump-shaped, with its unique peak is reached at, $\hat{z} \equiv Z(\hat{\psi}/A) < \bar{z}$, where

$$\mathcal{E}_{s(\zeta-1)/\zeta}(\hat{z}) = 0 \Leftrightarrow \frac{\hat{z}\zeta'(\hat{z})}{\zeta(\hat{z})} = [\zeta(\hat{z})-1]^2 \Leftrightarrow \mathcal{E}_\ell\left(\frac{\hat{\psi}}{A}\right) = 0 \Leftrightarrow \rho\left(\frac{\hat{\psi}}{A}\right)\sigma\left(\frac{\hat{\psi}}{A}\right) = 1.$$

A2+A3 are sufficient but not necessary for being hump-shaped.

Corollary of Proposition 4: Employments across active firms are

- decreasing in ψ, if $\hat{\psi} < \underline{\psi} \Leftrightarrow A < \underline{\psi}/Z^{-1}(\hat{z})$, which is possible only if $\underline{\psi} > 0$.
- increasing in ψ if $\psi_c < \hat{\psi} \Leftrightarrow F/E = \pi(\psi_c/A) > \pi(\hat{\psi}/A) = \pi(Z^{-1}(\hat{z}))$;

This occurs when the overhead/market size ratio is sufficiently high.

- hump-shaped in ψ if $\underline{\psi} < \hat{\psi} < \psi_c \Leftrightarrow F/E = \pi(\psi_c/A) < \pi(\hat{\psi}/A) = \pi(Z^{-1}(\hat{z}))$ & $A > \underline{\psi}/Z^{-1}(\hat{z})$.

Employments are decreasing among the most productive firms.

Proposition 5: Suppose that A2 and the strong A3 hold, so that $0 < \rho(\psi/A) < 1$ and $\rho(\psi/A)$ is strictly increasing. Then, $\rho(\psi/A)$ is strictly log-supermodular for all $\psi/A < \bar{z}$ with a sufficiently small $\bar{z}$.

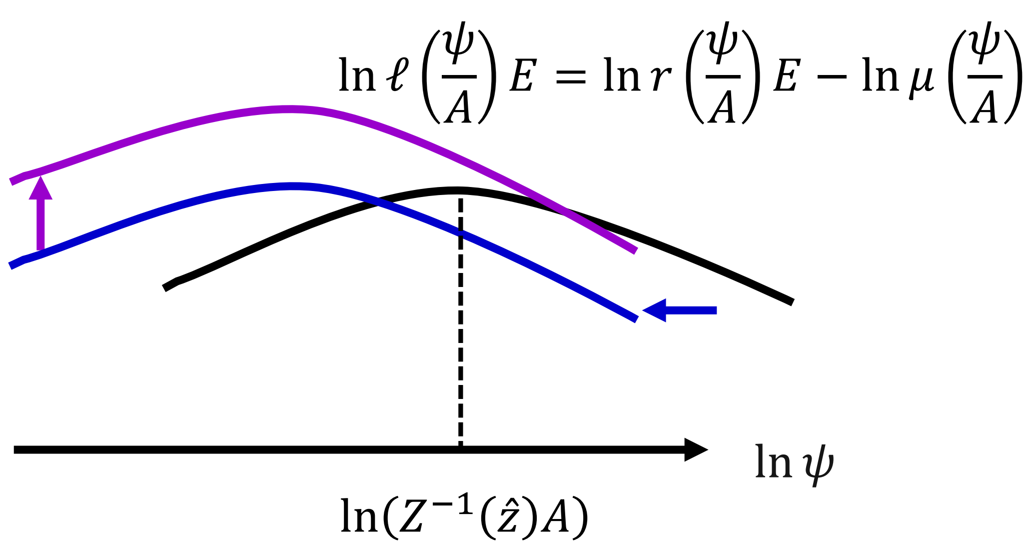
Employment Function: $\ell(\psi/A)E = r(\psi/A)E/\mu(\psi/A)$

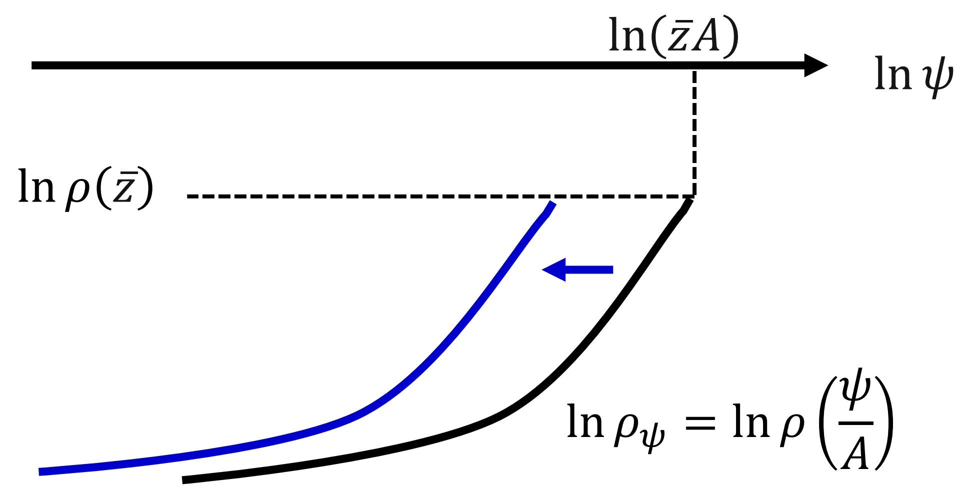
- *Hump-shaped* in ψ *under A2 and weak A3.*
  → $A\downarrow$ shifts up (down) for a low (high) ψ with $A\downarrow$
- Strictly log-submodular *under Weak A3*
  for $A\downarrow$ with a fixed E; for $A\downarrow$ caused by $E\uparrow$

Single-crossing even with a fixed E

Pass-Through Rate Function: $\rho_\psi = \rho(\psi/A)$

- $\rho(\psi/A) < 1$ *under A2, hence it cannot be* strictly log-supermodular for a higher range of ψ/A
- Strictly increasing in ψ *under Strong A3*
- Strictly log-supermodular for a lower range of ψ/A *under A2 and Strong A3* $\Rightarrow A\downarrow$ shifts up with a steeper slope at each ψ *with a small enough* $\bar{z}$*.*





In summary, more competitive pressures ($A\downarrow$)

- $\mu(\psi/A)\downarrow$ under A2 & $\rho(\psi/A)\uparrow$ under strong A3
- Profit, Revenue, Employment become more concentrated among the most productive.

5. Heterogenous Firms undre H.S.A.: Comparative Statics

5.1. Effects of F_e, E, and F on ψ_c and A

Proposition 6:

$$\begin{bmatrix} d\ln A \\ d\ln \psi_c \end{bmatrix} = \frac{\mathbb{E}_1(\pi)}{\mathbb{E}_1(\ell)} \begin{bmatrix} 1-f_x & f_x \\ 1-f_x & f_x-\delta \end{bmatrix} \begin{bmatrix} d\ln(F_e/E) \\ d\ln(F/E) \end{bmatrix}$$

where

$$\frac{\mathbb{E}_1(\pi)}{\mathbb{E}_1(\ell)} = \frac{1}{\mathbb{E}_\pi(\sigma)-1} = \{\mathbb{E}_r[\mu^{-1}]\}^{-1} - 1 = \mathbb{E}_\ell(\mu) - 1 > 0;$$

The average profit/the average labor cost ratio among the active firms

$$f_x \equiv \frac{FG(\psi_c)}{F_e + FG(\psi_c)} = \frac{\pi(\psi_c/A)}{\mathbb{E}_1(\pi)} < 1;$$

The share of the overhead in the total expected fixed cost = the profit of the cut-off firm relative to the average profit among the active firms

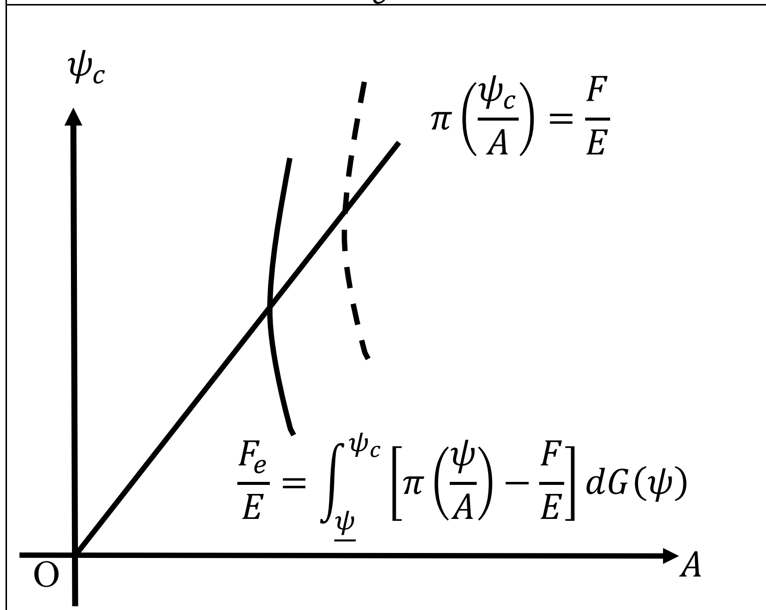
$$\delta \equiv \frac{\mathbb{E}_\pi(\sigma)-1}{\sigma(\psi_c/A)-1} = \frac{\pi(\psi_c/A)}{\ell(\psi_c/A)} \frac{\mathbb{E}_1(\ell)}{\mathbb{E}_1(\pi)} \equiv f_x \frac{\mathbb{E}_1(\ell)}{\ell(\psi_c/A)} > 0.$$

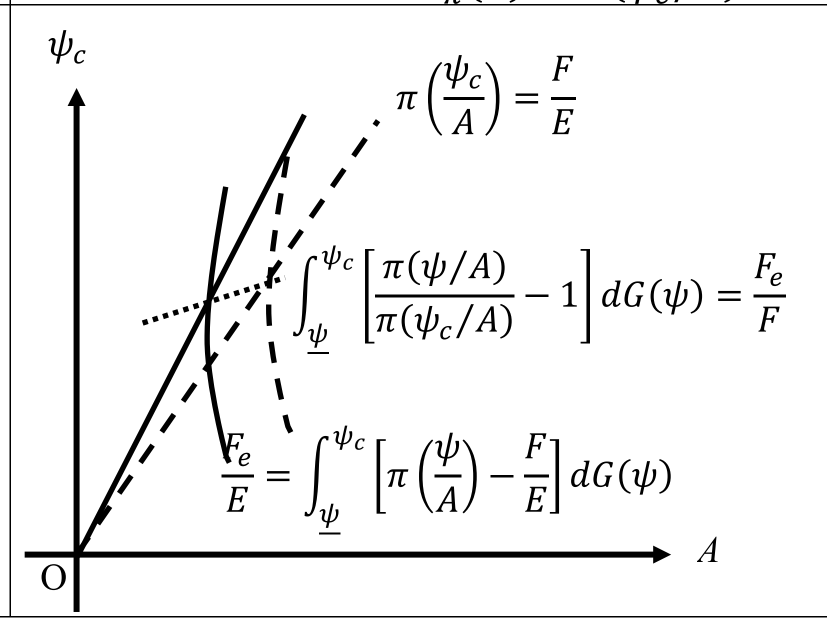
The profit/labor cost ratio of the cut-off firm to the average profit/average labor cost ratio among the active firms.

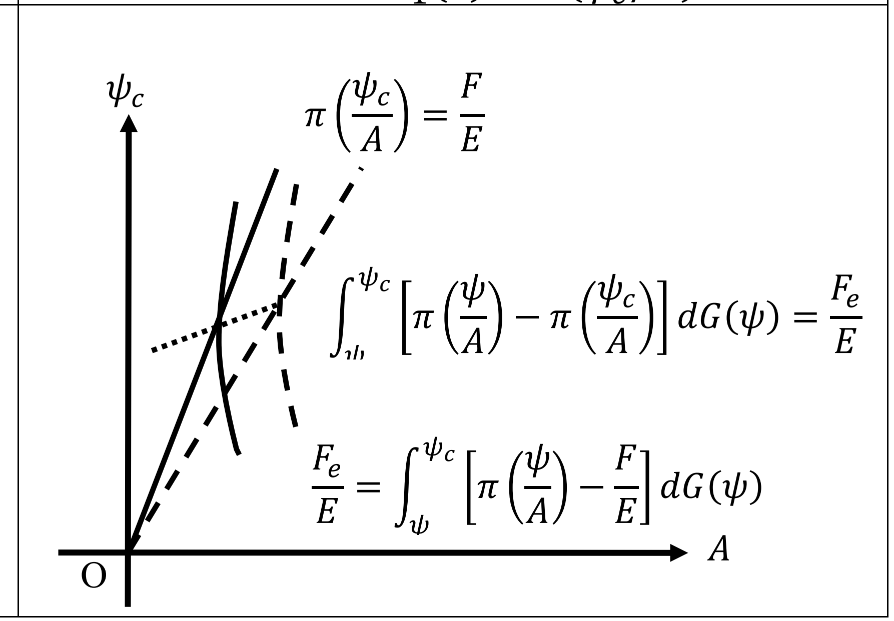
Corollary of Proposition 6

	A	ψ_c/A	ψ_c	
F_e	$\frac{d\ln A}{d\ln F_e} > 0$	$\frac{d\ln(\psi_c/A)}{d\ln F_e} = 0$	$\frac{d\ln \psi_c}{d\ln F_e} > 0$	
E	$\frac{d\ln A}{d\ln E} < 0$	$\frac{d\ln(\psi_c/A)}{d\ln E} > 0$	$\frac{d\ln \psi_c}{d\ln E} < 0 \Leftrightarrow \mathbb{E}_\pi(\sigma) < \sigma\left(\frac{\psi_c}{A}\right)$,	satisfied globally if $\sigma'(\cdot) > 0$, i.e., under A2.
F	$\frac{d\ln A}{d\ln F} > 0$	$\frac{d\ln(\psi_c/A)}{d\ln F} < 0$	$\frac{d\ln \psi_c}{d\ln F} > 0 \Leftrightarrow \mathbb{E}_1(\ell) < \ell\left(\frac{\psi_c}{A}\right)$,	satisfied globally if $\ell'(\cdot) > 0$.

$F_e \downarrow$	$E \uparrow$ under A2 $\Leftrightarrow \mathbb{E}_\pi(\sigma) < \sigma(\psi_c/A)$	$F \downarrow$ under $\mathbb{E}_1(\ell) < \ell(\psi_c/A)$







5.2. Market Size Effect on Profit, $\Pi_\psi \equiv \pi(\psi/A)E$ and Revenue, $R_\psi \equiv r(\psi/A)E$ (Proposition 7)

7a: Under **A2,** there exists a unique $\psi_0 \in (\underline{\psi}, \psi_c)$ such that $\sigma\left(\frac{\psi_0}{A}\right) = \mathbb{E}_\pi(\sigma)$ with

$$\frac{d\ln\Pi_\psi}{d\ln E} > 0 \Leftrightarrow \sigma\left(\frac{\psi}{A}\right) < \mathbb{E}_\pi(\sigma) \text{ for } \psi \in \left(\underline{\psi}, \psi_0\right),$$

and

$$\frac{d\ln\Pi_\psi}{d\ln E} < 0 \Leftrightarrow \sigma\left(\frac{\psi}{A}\right) > \mathbb{E}_\pi(\sigma) \text{ for } \psi \in (\psi_0, \psi_c).$$

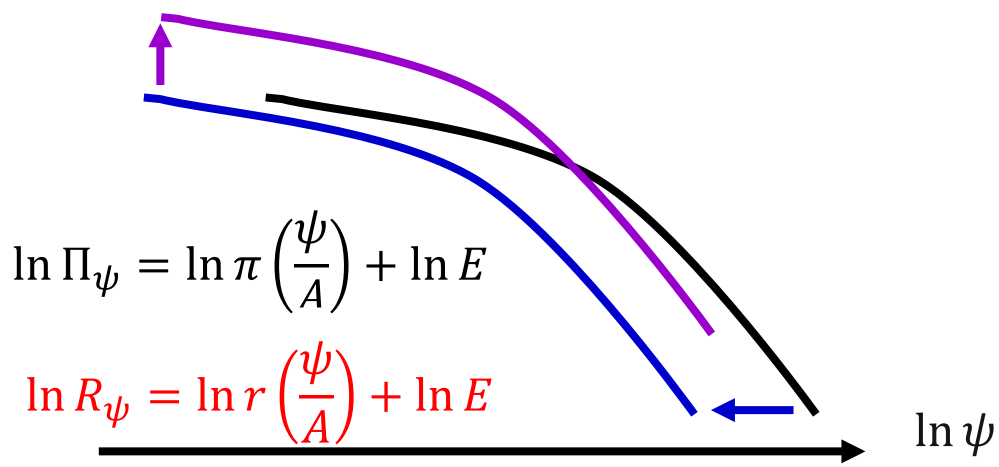
7b: Under **A2 and the weak A3,** there exists $\psi_1 > \psi_0$, such that

$$\frac{d\ln R_\psi}{d\ln E} > 0 \text{ for } \psi \in \left(\underline{\psi}, \psi_1\right).$$

Furthermore, $\psi_1 \in (\psi_0, \psi_c)$ and

$$\frac{d\ln R_\psi}{d\ln E} < 0 \text{ for } \psi \in (\psi_1, \psi_c),$$

for a sufficiently small F.



In short, more productive firms expand in absolute terms, while less productive firms shrink.

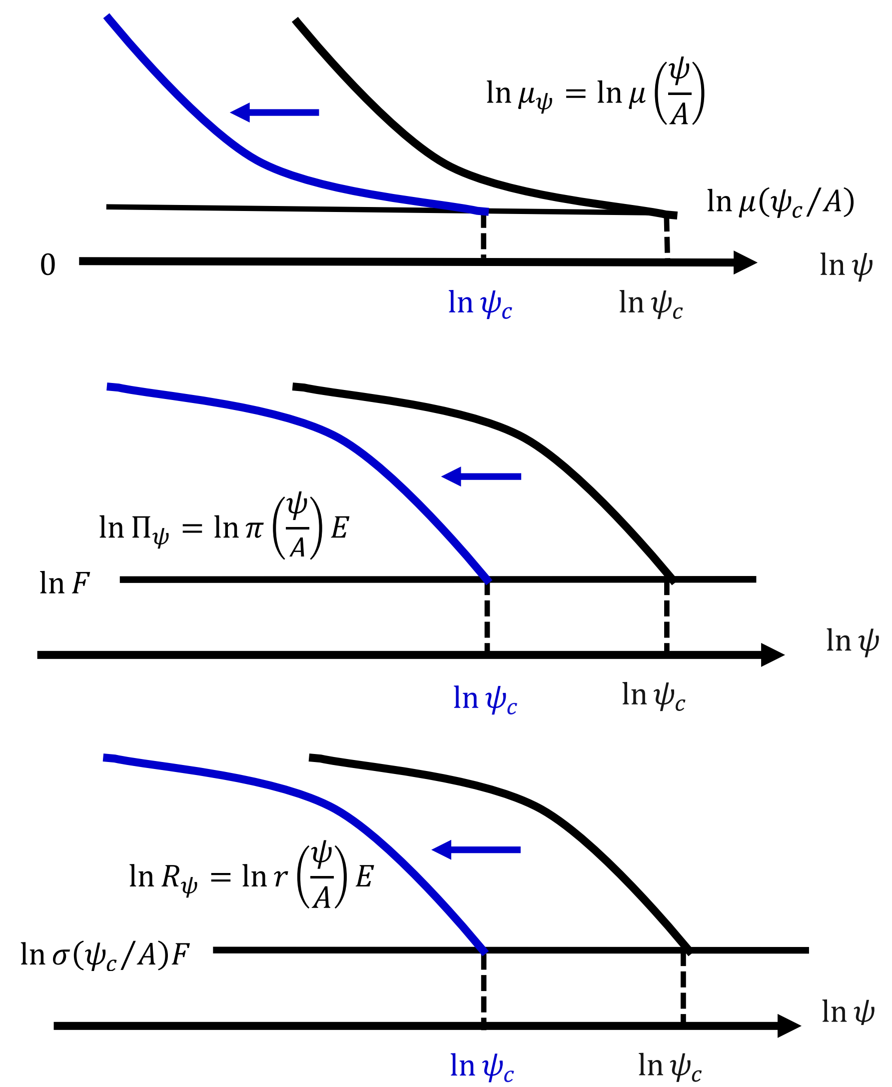
By putting together the main implications of Propositions 2, 3, 6, and 7

$F_e \downarrow$ under A2 and the weak A3

$A \downarrow$, $\psi_c \downarrow$ with ψ_c/A unchanged

The cutoff firms before the change and the cutoff firms after the change have

- the same markup rate $\mu(\psi_c/A)$
- the same profit $\pi(\psi_c/A)E = F$
- the same revenue, $r(\psi_c/A)E$



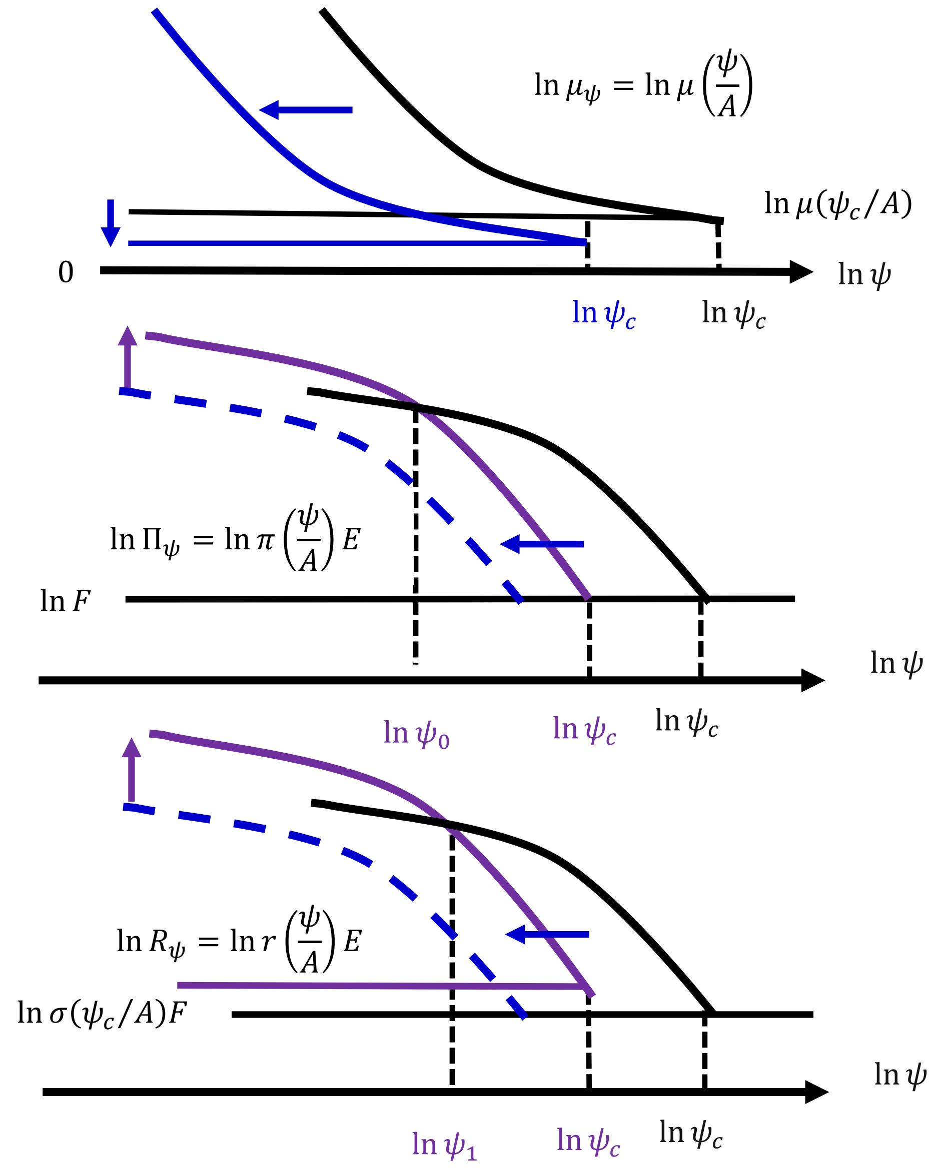
$E\uparrow$ under A2 and the weak A3

$A\downarrow,\ \psi_c\downarrow$ with $\psi_c/A\uparrow$ and $\sigma(\psi_c/A)\uparrow$

Compared to the cutoff firms before the change, the cutoff firms after the change have

- a lower markup rate, $\mu(\psi_c/A)\downarrow$
- the same profit, $\pi(\psi_c/A)E = F$.
- a higher revenue, $r(\psi_c/A)E = \sigma(\psi_c/A)F\uparrow$

Profits up (down) for firms with $\psi < (>)\psi_0$;
Revenues up (down) for firms with $\psi < (>)\psi_1$ for a sufficiently small F.

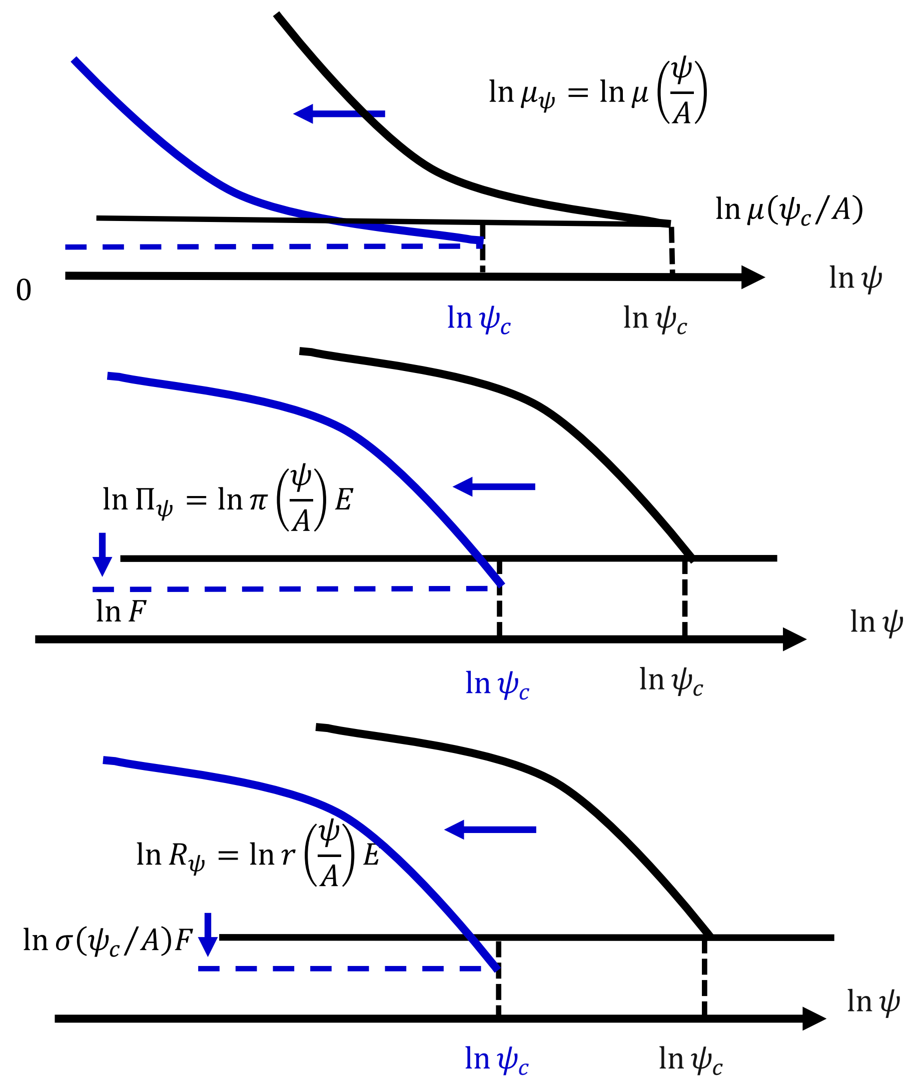


$F\downarrow$ under A2 and the weak A3 with $\ell'(\cdot) > 0$

$A\downarrow,\ \psi_c\downarrow$ with $\psi_c/A\uparrow$ and $\sigma(\psi_c/A)\uparrow$

Compared to the cutoff firms before the change, the cutoff firms after the change have

- a lower markup rate, $\mu(\psi_c/A)\downarrow$
- a lower profit, $\pi(\psi_c/A)E = F\downarrow$.
- a lower revenue, $r(\psi_c/A)E = \sigma(\psi_c/A)F\downarrow$.



5.3. The Composition Effect: Average Markup and Pass-Through Rates and P/A.

- Under A2, $A \downarrow$ causes $\mu(\psi/A) \downarrow$ for each ψ, but distribution shifts toward low-ψ firms with higher $\mu(\psi/A)$.
- Under strong A3, $A \downarrow$ causes $\rho(\psi/A) \uparrow$ for each ψ, but distribution shifts toward low-ψ firms with lower $\rho(\psi/A)$.

Proposition 8: *Assume that $\mathcal{E}_g'(\cdot)$ does not change its sign and $\underline{\psi} = 0$. Consider a shock to F_e, E, and/or F, which affects competitive pressures, i.e., $dA \neq 0$. Then, the response of any weighted generalized mean of any monotone function, $f(\psi/A) > 0$, defined by*

$$I \equiv \mathcal{M}^{-1}\left(\mathbb{E}_w\big(\mathcal{M}(f)\big)\right)$$

with a monotone transformation $\mathcal{M}: \mathbb{R}_+ \to \mathbb{R}$ and a weighting function, $w(\psi/A) > 0$, satisfies:

	$f'(\cdot) > 0$	$f'(\cdot) = 0$	$f'(\cdot) < 0$
$\mathcal{E}_g'(\cdot) > 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \geq 0 \Rightarrow \frac{d\ln I}{d\ln A} > 0$	$\frac{d\ln I}{d\ln A} = 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \geq 0 \Rightarrow \frac{d\ln I}{d\ln A} < 0$
$\mathcal{E}_g'(\cdot) = 0$ (Pareto)	$\frac{d\ln(\psi_c/A)}{d\ln A} \gtreqless 0 \Leftrightarrow \frac{d\ln I}{d\ln A} \gtreqless 0$	$\frac{d\ln I}{d\ln A} = 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \gtreqless 0 \Leftrightarrow \frac{d\ln I}{d\ln A} \lesseqgtr 0$
$\mathcal{E}_g'(\cdot) < 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \leq 0 \Rightarrow \frac{d\ln I}{d\ln A} < 0$	$\frac{d\ln I}{d\ln A} = 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \leq 0 \Rightarrow \frac{d\ln I}{d\ln A} > 0$

Moreover, if $\mathcal{E}_g'(\cdot) = \frac{d\ln(\psi_c/A)}{d\ln A} = 0$, $d\ln I/d\ln A = 0$ for any $f(\psi/A)$, monotonic or not. Furthermore, $\mathcal{E}_g'(\cdot)$ can be replaced with $\mathcal{E}_G'(\cdot)$ in all the above statements for $w(\psi/A) = 1$, i.e., the unweighted averages.

$I \equiv \mathcal{M}^{-1}\left(\mathbb{E}_w\big(\mathcal{M}(f)\big)\right)$ can be any Hölder mean, including the arithmetic, $I = \mathbb{E}_w(f)$, the geometric, $I = \exp[\mathbb{E}_w(\ln f)]$, and the harmonic,$I = \left(\mathbb{E}_w(f^{-1})\right)^{-1}$, and the weight function, $w(\psi/A)$, can be profit, revenue, and employment, or unweighted.

Corollary 1 of Proposition 8

a) Entry Cost: $f'(\cdot)\mathcal{E}_g'(\cdot) \gtreqless 0 \Leftrightarrow \frac{d\ln I}{d\ln F_e} = \frac{d\ln I}{d\ln A}\frac{d\ln A}{d\ln F_e} \gtreqless 0.$

b) Market Size: *If* $\mathcal{E}_g'(\cdot) \leq 0$, *then,* $f'(\cdot) \gtreqless 0 \Longrightarrow \frac{d\ln I}{d\ln E} = \frac{d\ln I}{d\ln A}\frac{d\ln A}{d\ln E} \gtreqless 0.$

c) Overhead Cost: *If* $\mathcal{E}_g'(\cdot) \leq 0$, *then,* $f'(\cdot) \gtreqless 0 \Longrightarrow \frac{d\ln I}{d\ln F} = \frac{d\ln I}{d\ln A}\frac{d\ln A}{d\ln F} \lesseqgtr 0.$

Furthermore, $\mathcal{E}_g'(\cdot)$ *can be replaced with* $\mathcal{E}_G'(\cdot)$ *for* $w(\psi/A) = 1$, *i.e., the unweighted averages.*

For the entry cost, $\frac{d\ln(\psi_c/A)}{d\ln A} = 0.$

- $\mathcal{E}_g'(\cdot) > 0$; sufficient & necessary for the composition effect to dominate:
  - The average markup & pass-through rates move in the *opposite* direction from the firm-level rates
- $\mathcal{E}_g'(\cdot) = 0$ (Pareto); a knife-edge. $A \downarrow$ → no change in average markup and pass-through.
- $\mathcal{E}_g'(\cdot) < 0$; sufficient & necessary for the procompetitive effect to dominate:
  The average markup & pass-through rates move in the *same* direction from the firm-level rates

For market size and the overhead cost, $\frac{d\ln(\psi_c/A)}{d\ln A} < 0$

- $\mathcal{E}_g'(\cdot) > 0$; necessary for the composition effect to dominate:
- $\mathcal{E}_g'(\cdot) \leq 0$; sufficient for the procompetitive effect to dominate:

The Composition Effect: Impact on P/A

$$\ln\left(\frac{A}{cP}\right) = \mathbb{E}_r[\Phi \circ Z]$$

$$\zeta'(\cdot) \gtreqless 0 \implies \Phi'(\cdot) \lesseqgtr 0 \Leftrightarrow \Phi \circ Z'(\cdot) \lesseqgtr 0$$

Corollary 2 of Proposition 8: *Assume* $\underline{\psi} = 0$*, and neither* $\zeta'(\cdot)$ *nor* $\mathcal{E}_g'(\cdot)$ *change the signs. Consider a shock to* F_e, E*, and/or* F*, which affects competitive pressures, i.e.,* $dA \neq 0$*. Then, the response of* P/A *satisfies:*

	$\zeta'(\cdot) > 0$ (A2)	$\zeta'(\cdot) = 0$ (CES)	$\zeta'(\cdot) < 0$
$\mathcal{E}_g'(\cdot) > 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \geq 0 \implies \frac{d\ln(P/A)}{d\ln A} > 0$	$\frac{d\ln(P/A)}{d\ln A} = 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \geq 0 \implies \frac{d\ln(P/A)}{d\ln A} < 0$
$\mathcal{E}_g'(\cdot) = 0$ (Pareto)	$\frac{d\ln(\psi_c/A)}{d\ln A} \gtreqless 0 \Leftrightarrow \frac{d\ln(P/A)}{d\ln A} \gtreqless 0$	$\frac{d\ln(P/A)}{d\ln A} = 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \gtreqless 0 \Leftrightarrow \frac{d\ln(P/A)}{d\ln A} \lesseqgtr 0$
$\mathcal{E}_g'(\cdot) < 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \leq 0 \implies \frac{d\ln(P/A)}{d\ln A} < 0$	$\frac{d\ln(P/A)}{d\ln A} = 0$	$\frac{d\ln(\psi_c/A)}{d\ln A} \leq 0 \implies \frac{d\ln(P/A)}{d\ln A} > 0$

5.4 Comparative Statics on $M, MG(\psi_c)$, and TFP.

Proposition 9: *Assume that $\mathcal{E}'_G(\cdot)$ does not change its sign and $\underline{\psi} = 0$. Consider a shock to F_e, F, and/or E, which affects competitive pressures, i.e., $dA \neq 0$. Then, the response of the mass of active firms, $MG(\psi_c)$, is as follows:*

$$If\ \mathcal{E}'_G(\cdot) > 0, \qquad \frac{d\ln(\psi_c/A)}{d\ln A} \geq 0 \Rightarrow \frac{d\ln[MG(\psi_c)]}{d\ln A} > 0;$$

$$If\ \mathcal{E}'_G(\cdot) = 0, \qquad \frac{d\ln(\psi_c/A)}{d\ln A} \gtreqless 0 \Leftrightarrow \frac{d\ln[MG(\psi_c)]}{d\ln A} \gtreqless 0;$$

$$If\ \mathcal{E}'_G(\cdot) < 0, \qquad \frac{d\ln(\psi_c/A)}{d\ln A} \leq 0 \Rightarrow \frac{d\ln[MG(\psi_c)]}{d\ln A} < 0.$$

Corollary 1 of Proposition 9

a) Entry Cost: $\mathcal{E}'_G(\cdot) \gtreqless 0 \Leftrightarrow \frac{d\ln[MG(\psi_c)]}{d\ln F_e} = \frac{d\ln[MG(\psi_c)]}{d\ln A}\frac{d\ln A}{d\ln F_e} \gtreqless 0.$

b) Market Size: $\mathcal{E}'_G(\cdot) \leq 0 \Rightarrow \frac{d\ln[MG(\psi_c)]}{d\ln E} = \frac{d\ln[MG(\psi_c)]}{d\ln A}\frac{d\ln A}{d\ln E} > 0.$

c) Overhead Cost: $\mathcal{E}'_G(\cdot) \leq 0 \Rightarrow \frac{d\ln[MG(\psi_c)]}{d\ln F} = \frac{d\ln[MG(\psi_c)]}{d\ln A}\frac{d\ln A}{d\ln F} < 0.$

For a decline in the entry cost,
$\mathcal{E}'_g(\cdot) > 0$ *sufficient & necessary* for $MG(\psi_c)$ ↓; $\mathcal{E}'_g(\cdot) = 0$, no effect; $\mathcal{E}'_g(\cdot) < 0$; *sufficient & necessary* for $MG(\psi_c)$ ↑
For market size and the overhead cost
$\mathcal{E}'_g(\cdot) > 0$ *necessary* for $MG(\psi_c)$ ↓; $\mathcal{E}'_g(\cdot) \leq 0$ *sufficient* for $MG(\psi_c)$ ↑

Impact of Competitive Pressures on Unit Cost/TFP

By combining Corollary 2 of Proposition 8 and Corollary 1 of Proposition,

Corollary 2 of Proposition 9: *Assume* $\underline{\psi} = 0$, *and neither* $\zeta'(\cdot)$ *nor* $\mathcal{E}_g'(\cdot)$ *change the signs. Consider a shock to* F_e, L, *and/or* F, *which affects competitive pressures, i.e.,* $dA \neq 0$. *Then, the response of* P *satisfies:*

	$\zeta'(\cdot) > 0$ (A2)	$\zeta'(\cdot) = 0$ (CES)	$\zeta'(\cdot) < 0$
$\mathcal{E}_g'(\cdot) > 0$	$\frac{d \ln P}{d \ln A} > 1 \; for \; F_e$	$\frac{d \ln P}{d \ln A} = 1$	?
$\mathcal{E}_g'(\cdot) = 0$ (Pareto)	$\frac{d \ln P}{d \ln A} = 1 \; for \; F_e$ $0 < \frac{d \ln P}{d \ln A} < 1 \; for \; F \; or \; E;$	$\frac{d \ln P}{d \ln A} = 1$	$\frac{d \ln P}{d \ln A} = 1 \; for \; F_e$ $\frac{d \ln P}{d \ln A} > 1 \; for \; F \; or \; E$
$\mathcal{E}_g'(\cdot) < 0$	$0 < \frac{d \ln P}{d \ln A} < 1$	$\frac{d \ln P}{d \ln A} = 1$	$\frac{d \ln P}{d \ln A} > 1$

Limit Case of $F \to 0$ with $\bar{z} < \infty$

Cutoff Rule:	$\pi\left(\frac{\psi_c}{A}\right) = 0 \Leftrightarrow \frac{\psi_c}{A} = \bar{z} = \pi^{-1}(0)$
Free Entry Condition:	$\frac{F_e}{E} = \int_{\underline{\psi}}^{\psi_c} \pi\left(\bar{z}\frac{\psi}{\psi_c}\right) dG(\psi) = \int_{\underline{\psi}}^{\bar{z}A} \pi\left(\frac{\psi}{A}\right) dG(\psi).$

A & ψ_c: uniquely determined as C^2 functions of F_e/E with the interior solution, $0 < G(\psi_c) < 1$ for $0 < \frac{F_e}{E} < \int_{\underline{\psi}}^{\bar{\psi}} \pi\left(\bar{z}\frac{\psi}{\bar{\psi}}\right) dG(\psi)$.

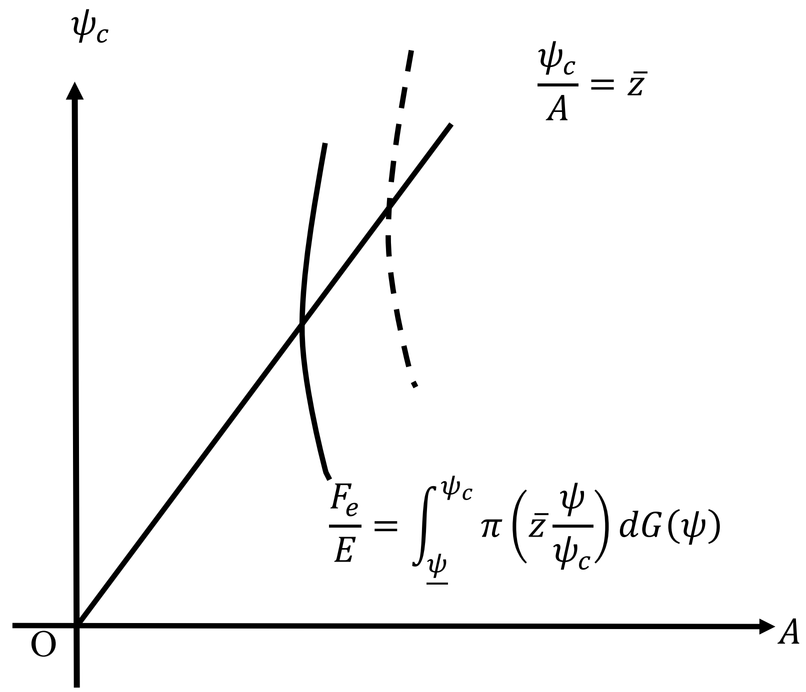
$$\frac{d\psi_c}{\psi_c} = \frac{dA}{A} = \frac{\mathbb{E}_1(\pi)}{\mathbb{E}_1(\ell)} \frac{d(F_e/E)}{F_e/E}.$$

$E \uparrow$ is isomorphic to $F_e \downarrow$.

For $I \equiv \mathcal{M}^{-1}\left(\mathbb{E}_w(\mathcal{M}(f))\right)$

$$f'(\cdot)\mathcal{E}_g'(\cdot) \gtreqless 0 \Leftrightarrow \frac{d\ln I}{d\ln(F_e/E)} = \frac{d\ln I}{d\ln A}\frac{d\ln A}{d\ln(F_e/E)} \gtreqless 0.$$

$$\mathcal{E}_g'(\cdot) \gtreqless 0 \Leftrightarrow \frac{d\ln[MG(\psi_c)]}{d\ln(F_e/E)} = \frac{d\ln[MG(\psi_c)]}{d\ln A}\frac{d\ln A}{d\ln(F_e/E)} \gtreqless 0.$$



$F_e/E \downarrow$ for $F \to 0$ with $\bar{z} < \infty$ under A2 and the weak A3

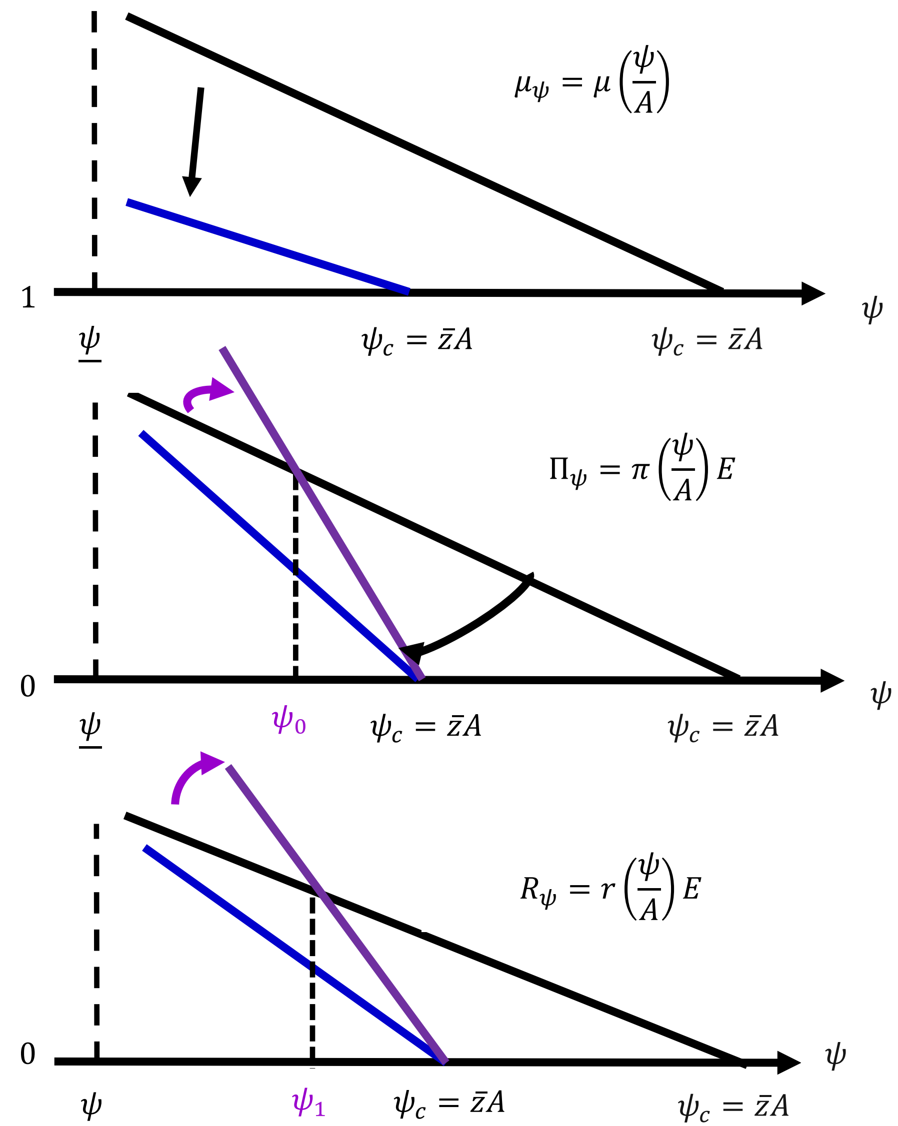
$A \downarrow, \psi_c \downarrow$ with $\psi_c/A = \bar{z}$ unchanged.

The cutoff firms always (i.e., both before and after the change) have

- $\mu(\psi_c/A) = 1$
- $\pi(\psi_c/A)E = 0.$
- $r(\psi_c/A)E = 0.$

Profits up (down) for firms with $\psi < (>)\psi_0$;
Revenues up (down) for firms with $\psi < (>)\psi_1$.

In the middle and the lower panels,
Blue : the effects of $F_e/E \downarrow$ due to $F_e \downarrow$
Purple: the effects of $F_e/E \downarrow$ due to $E \uparrow$



6. Sorting of Heterogeneous Firms Across Multiple Markets

6.1. A Multi-Market Extension: *J* markets, $j = 1,2,\ldots,J$, with market size E_j.

Possible Interpretations

- Households with the total spending, E, with Cobb-Douglas, $\sum_{j=1}^{J}\beta_j \ln X_j$ with $\sum_{j=1}^{J}\beta_j = 1$. Then, $E_j = \beta_j E$.
- J types of consumers, with E_j the total spending of type-j consumers. "Types" can be "tastes" or "locations", etc.

Assume

- Market size is the only exogenous source of heterogeneity across markets: Index them as $E_1 > E_2 >, \ldots, > E_J > 0$.
- Numeraire, "labor," is fully mobile, equalizing its price across the markets. If markets are spatially separated, this may be unrealistic but innocuous; the factor price difference across markets affects the market choice of all firms equally, regardless of their productivity; it doesn't affect their sorting across markets.
- Firm's marginal cost, ψ, is independent of the market it chooses.
  - Each firm pays $F_e > 0$ to draw its marginal cost $\psi \sim G(\psi)$.
  - Knowing its ψ, each firm decides which market to enter with an overhead cost, $F > 0$, or exit without producing.
  - Firms sell their products at the profit-maximizing prices in the market they enter.

Equilibrium Condition:

$$F_e = \int_{\underline{\psi}}^{\overline{\psi}} \max\{\Pi_\psi - F, 0\}\, dG(\psi) = \int_{\underline{\psi}}^{\overline{\psi}} \max\left\{\max_{1\le j\le J}\{\Pi_{j\psi}\} - F, 0\right\} dG(\psi)$$

$$\text{where} \quad \Pi_{j\psi} \equiv \frac{s\left(Z(\psi/A_j)\right)}{\zeta\left(Z(\psi/A_j)\right)} E_j \equiv \frac{r(\psi/A_j)}{\sigma(\psi/A_j)} E_j = \pi\left(\frac{\psi}{A_j}\right) E_j$$

6.2. Positive Assortative Matching Between Firm Productivity and Market Size

Proposition 10: Equilibrium Characterization under A2

Larger markets are more competitive:

$$0 < A_1 < A_2 < \cdots < A_J < \infty, \text{where } M \int_{\psi_{j-1}}^{\psi_j} r\left(\frac{\psi}{A_j}\right) dG(\psi) = 1.$$

Note: Because $\pi(\cdot)$ is strictly decreasing, this implies $\pi(\psi/A_1) < \pi(\psi/A_2) < \cdots < \pi(\psi/A_J)$ for all ψ.

More productive firms self-select into larger markets (Positive Assortative Matching)

Firms with $\psi \in (\psi_{j-1}, \psi_j)$ enter market-j and those with $\psi \in (\psi_J, \infty)$ do not enter any market, where

$$0 \leq \underline{\psi} = \psi_0 < \psi_1 < \psi_2 < \cdots < \psi_J < \overline{\psi} \leq \infty \quad \text{where } \frac{\pi(\psi_j/A_j)E_j}{\pi(\psi_j/A_{j+1})E_{j+1}} = 1 \text{ for } 1 \leq j \leq J-1; \quad \pi\left(\frac{\psi_J}{A_J}\right) E_J \equiv F$$

Note: ψ_j-firms are indifferent btw entering Market-j & entering Market-$(j+1)$.

Free Entry Condition: $$\sum_{j=1}^{J} \int_{\psi_{j-1}}^{\psi_j} \left\{\pi\left(\frac{\psi}{A_j}\right) E_j - F\right\} dG(\psi) = F_e$$

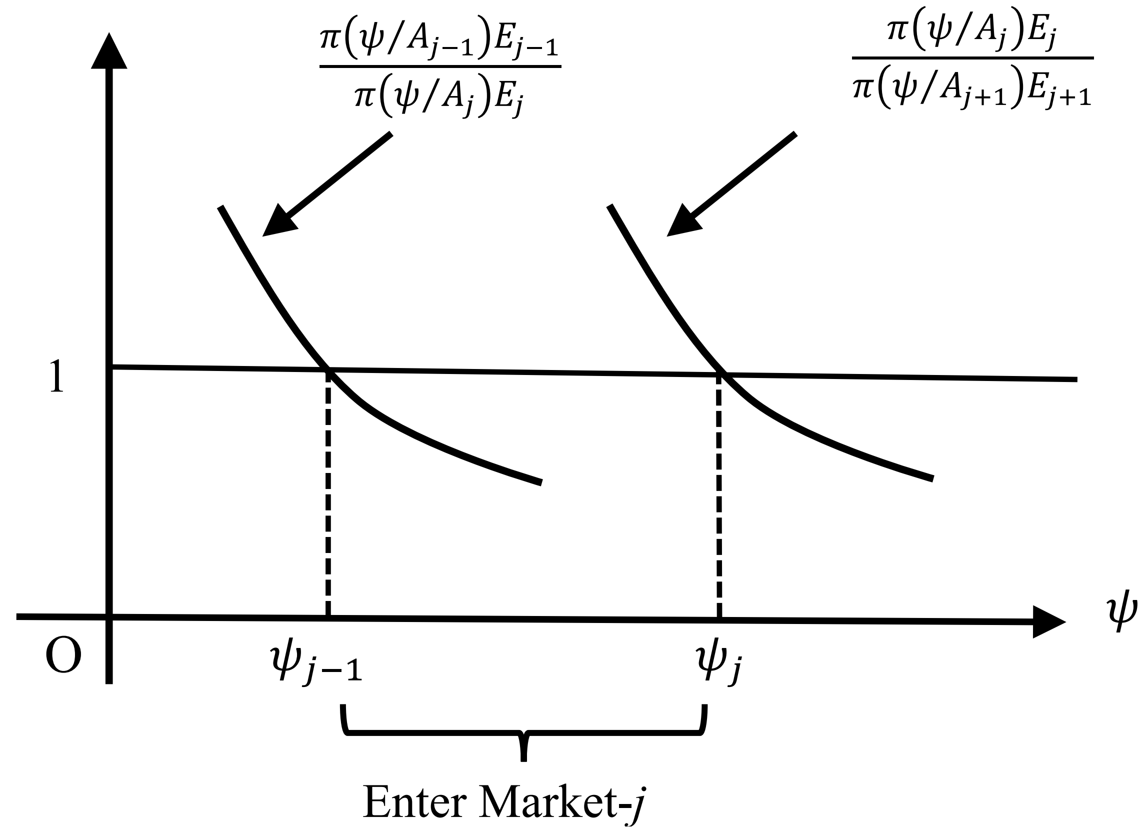
Mass of Firms in Market-*j*: $$M[G(\psi_j) - G(\psi_{j-1})] > 0$$

Logic Behind Sorting

$E_j > E_{j+1} \Longrightarrow A_j < A_{j+1}$. Otherwise, no firm would enter $j+1$.

$\Longrightarrow \frac{\pi(\psi/A_j)E_j}{\pi(\psi/A_{j+1})E_{j+1}}$, strictly decreasing in ψ, due to strict log-submodularity of $\pi(\psi/A)$ in ψ and $1/A$ under **A2.**

$$\Longrightarrow \left[\frac{\Pi_{j\psi}}{\Pi_{(j+1)\psi}} = \frac{\pi(\psi/A_j)E_j}{\pi(\psi/A_{j+1})E_{j+1}} \gtreqless 1 \Leftrightarrow \psi \lesseqgtr \psi_j\right]$$



Under CES, $\frac{\pi(\psi/A_j)E_j}{\pi(\psi/A_{j+1})E_{j+1}}$ is independent of ψ.

$\Longrightarrow \frac{\Pi_{j\psi}}{\Pi_{(j+1)\psi}} = \frac{\pi(\psi/A_j)E_j}{\pi(\psi/A_{j+1})E_{j+1}} = 1$ in equilibrium.

$\Longrightarrow$ Firms indifferent across all markets.

$\Longrightarrow$ Distribution of firms across markets is indeterminate.

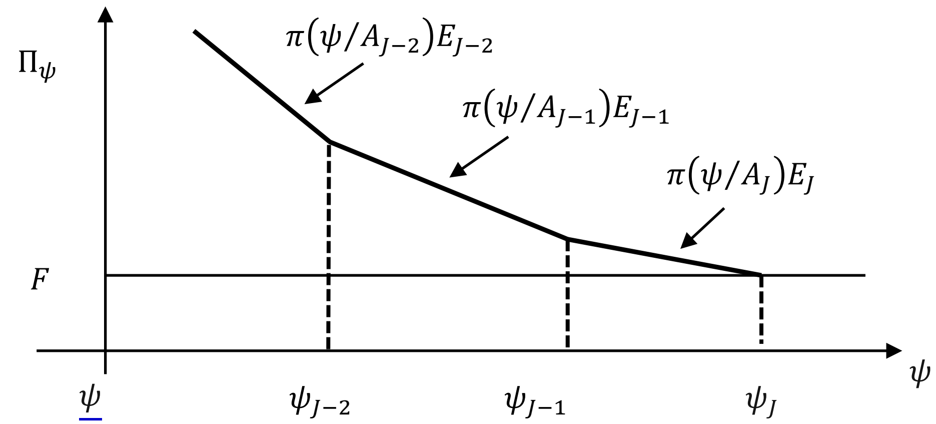
Our mechanism generates sorting through competitive pressures. As such,

- complementary to agglomeration-economies-based mechanisms offered by Gaubert (2018) and Davis-Dingel (2019)
- justifies the equilibrium selection criterion used by Baldwin-Okubo (2006), which use CES, as a limit argument.

6.3. Cross-Sectional, Cross-Market Implications:

Profits: Under A2

$$E_j > E_{j+1} \Rightarrow A_j < A_{j+1} \Rightarrow \left[\frac{\pi(\psi/A_j)E_j}{\pi(\psi/A_{j+1})E_{j+1}} \gtreqless 1 \Leftrightarrow \psi \lesseqgtr \psi_j\right]$$

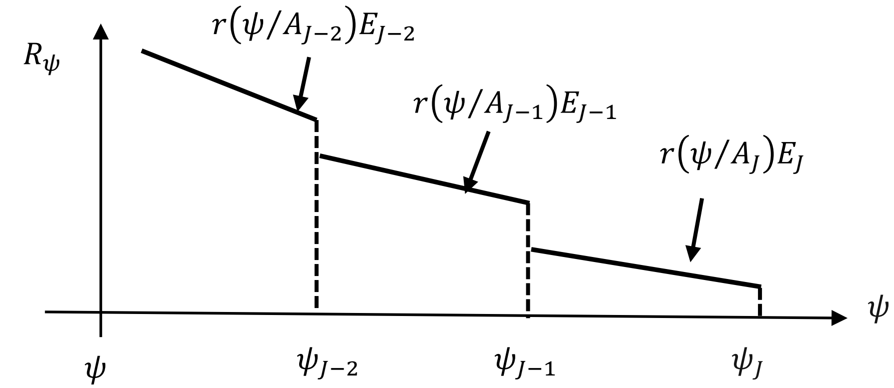


$\Pi_\psi = \max_j \left\{\pi\left(\frac{\psi}{A_j}\right)E_j\right\}$, the upper-envelope of $\pi(\psi/A_j)E_j$, is continuous and decreasing in ψ, with the kinks at ψ_j.

Continuous, as the lower markup rate in j cancels out its larger market size, keeping ψ_j-firms indiffierent btw j & $j+1$.

Revenues: Under A2

$$\frac{r(\psi_j/A_j)E_j}{r(\psi_j/A_{j+1})E_{j+1}} = \frac{\sigma(\psi_j/A_j)\pi(\psi_j/A_j)E_j}{\sigma(\psi_j/A_{j+1})\pi(\psi_j/A_{j+1})E_{j+1}} = \frac{\sigma(\psi_j/A_j)}{\sigma(\psi_j/A_{j+1})} > 1$$

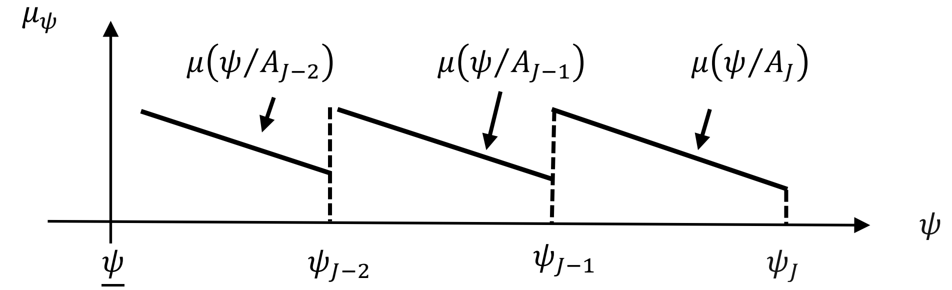


R_ψ: continuously decreasing in ψ within each market; jumps down at ψ_j.
With the markup rate lower in market-j, ψ_j-firms need to earn higher revenue to keep them indiffierent btw j & $j+1$.

Markup Rates: Under A2

$$E_j > E_{j+1} \Rightarrow A_j < A_{j+1} \Rightarrow \sigma\left(\frac{\psi_j}{A_j}\right) > \sigma\left(\frac{\psi_j}{A_{j+1}}\right) \Leftrightarrow \mu\left(\frac{\psi_j}{A_j}\right) < \mu\left(\frac{\psi_j}{A_{j+1}}\right)$$

μ_ψ: continuously decreasing in ψ within each market but jumps up at ψ_j.

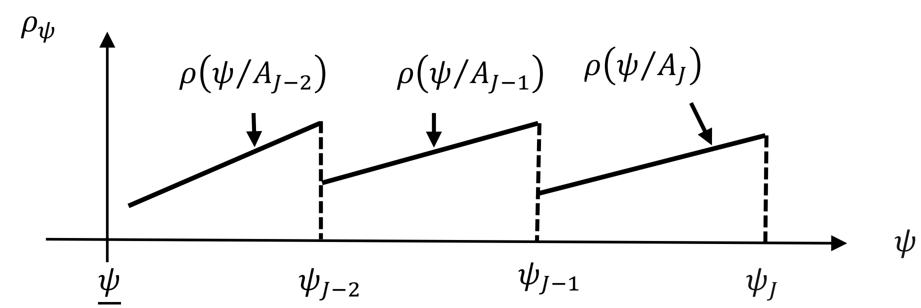


- The average markup rates may be *higher* in larger (and hence more competitive) markets.
- The average markup rates in all markets may *go up*, even if all markets become more competitive ($A_j \downarrow$).

Pass-Through Rates: Under A2 and the strong A3

$$E_j > E_{j+1} \Rightarrow A_j < A_{j+1} \Rightarrow \rho\left(\frac{\psi_j}{A_j}\right) > \rho\left(\frac{\psi_j}{A_{j+1}}\right)$$

ρ_ψ: continuously increasing in ψ within each market but jumps down at ψ_j.



- The average pass-through rates may be *lower* in larger (and hence more competitive) markets.
- The average pass-through rates in all markets go *down* even if all markets become more competitive ($A_j \downarrow$).

6.4. Average Markup and Pass-Through Rates Across Markets: The Composition Effect

Proposition 11a: *Suppose A2 and* $G(\psi) = \left(\psi/\overline{\psi}\right)^{\kappa}$. *There exists a sequence,* $E_1 > E_2 > \cdots > E_J > 0$, *such that, in equilibrium, any weighted generalized mean of* $f\left(\psi/A_j\right)$ *across firms operating at market-*j *are increasing (decreasing) in* j *even though* $f(\cdot)$ *is increasing (decreasing) and hence* $f\left(\psi/A_j\right)$ *is decreasing (increasing) in* j.

Corollary of Proposition 11a: An example with $G(\psi) = \left(\psi/\overline{\psi}\right)^{\kappa}$, such that the average markup rates are *higher* (and the average pass-through rates are *lower* under Strong A3) in larger markets.

Proposition 11b: *Suppose A2 and* $G(\psi) = \left(\psi/\overline{\psi}\right)^{\kappa}$. *Then, a change in* F_e *keeps*

i) *the ratios* $a_j \equiv \psi_{j-1}/\psi_j$ *and* $b_j \equiv \psi_j/A_j$

and

ii) *any weighted generalized mean of* $f\left(\psi/A_j\right)$ *across firms operating at market-*j, *for any weighting function* $w\left(\psi/A_j\right)$,

unchanged for all $j = 1,2, \dots, J$.

Corollary of Proposition 11b: $F_e \downarrow$ and $G(\psi) = \left(\psi/\overline{\psi}\right)^{\kappa}$ offers a knife-edge case, where the average markup and pass-through rates of all markets remain unchanged.

A caution against testing A2/A3 by comparing the average markup & pass-through rates across space and time.

7. International/Interregional Trade with Differential Market Access

Two Symmetric Markets, characterized by

The same market size E, "Labor" supplied at the same price (equal to one), the numeraire, ensuring the same level of competitive pressures, A.

- After paying F_e, & learning ψ_ω, firm ω can produce its product at home & sell to both markets.
  - The overhead cost, $F > 0$ and the marginal cost of selling to the home market, ψ_ω.
  - The overhead cost, $F > 0$ and the marginal cost of selling to the export market, $\tau\psi_\omega > \psi_\omega$. **Iceberg cost, $\boldsymbol{\tau > 1}$.**

Cutoff Rules: Firm ω sells to both markets iff $\psi_\omega \leq \psi_{xc} < \psi_c$; only to the home market iff $\psi_{xc} < \psi_\omega \leq \psi_c$, where

$$F \equiv \pi\left(\frac{\psi_c}{A}\right)E \equiv \pi\left(\frac{\tau\psi_{xc}}{A}\right)E.$$

Free-Entry Condition:

$$F_e = \int_{\underline{\psi}}^{\psi_c}\left[\pi\left(\frac{\psi}{A}\right)E - F\right]dG(\psi) + \int_{\underline{\psi}}^{\psi_{xc}}\left[\pi\left(\frac{\tau\psi}{A}\right)E - F\right]dG(\psi).$$

These two conditions jointly pin down the equilibrium value of $\psi_c \equiv \tau\psi_{xc} \equiv \pi^{-1}(F/E)A$ by:

$$\frac{F_e}{E} = \int_{\underline{\psi}}^{\psi_c}\left[\pi\left(\frac{\psi}{\psi_c}\pi^{-1}\left(\frac{F}{E}\right)\right) - \frac{F}{E}\right]dG(\psi) + \int_{\underline{\psi}}^{\psi_c/\tau}\left[\pi\left(\frac{\tau\psi}{\psi_c}\pi^{-1}\left(\frac{F}{E}\right)\right) - \frac{F}{E}\right]dG(\psi).$$

After solving for $\psi_c \equiv \tau\psi_{xc} \equiv \pi^{-1}(F/E)A$, the mass of entering firms, M, and hence those of active firms $MG(\psi_c)$, and of exporting firms, $MG(\psi_{xc})$, are pinned down by:

Adding-Up (Resource) Constraint:

$$M\left[\int_{\underline{\psi}}^{\psi_c} r\left(\frac{\psi}{A}\right) dG(\psi) + \int_{\underline{\psi}}^{\psi_{xc}} r\left(\frac{\tau\psi}{A}\right) dG(\psi)\right] = 1.$$

Proposition 12: The Effect of Globalization: A Reduction in $\tau > 1$.

- A decline in ψ_c and an increase in $\psi_{xc} = \psi_c/\tau$. → $G(\psi_c)$ falls, $G(\psi_{xc})$ rises, and $G(\psi_{xc})/G(\psi_c)$ rises.
- A decline in A and an increase in A/τ. →
    - $r(\psi_\omega/A)$ & $\pi(\psi_\omega/A)$ decline, $r(\tau\psi_\omega/A)$ & $\pi(\tau\psi_\omega/A)$ rise.
    - $\mu(\psi_\omega/A)$ declines and $\mu(\tau\psi_\omega/A)$ rises **under the 2nd law.**
    - $\rho(\psi_\omega/A)$ rises and $\rho(\tau\psi_\omega/A)$ declines **under the Strong 3rd law.**

Appendices

Symmetric H.S.A. with Gross Substitutes: An Alternative (Equivalent) Definition

Market Share of ω depends *solely* on its own quantity normalized by the *common* quantity aggregator

$$s_\omega \equiv \frac{p_\omega x_\omega}{\mathbf{px}} = \frac{\partial \ln X(\mathbf{x})}{\partial \ln x_\omega} = s^*\left(\frac{x_\omega}{A^*(\mathbf{x})}\right), \qquad \text{where} \quad \int_\Omega s^*\left(\frac{x_\omega}{A^*(\mathbf{x})}\right) d\omega \equiv 1.$$

- $s^*: \mathbb{R}_{++} \to \mathbb{R}_+$: **the market share function**, with $0 < \mathcal{E}_{s^*}(y_\omega) < 1$, where $y_\omega \equiv x_\omega / A^*$ is **the normalized quantity**
    - If $\bar{z} \equiv s^{*\prime}(0) = \lim_{y\to 0}[s^*(y)/y] < \infty$, $\bar{z}A(\mathbf{p})$ is the **choke price**.
- $A^* = A^*(\mathbf{x})$: the **common quantity aggregator** defined implicitly by **the adding up constraint** $\int_\Omega s^*(x_\omega/A^*)d\omega \equiv 1$. $A^*(\mathbf{x})$ linear homogenous in $\mathbf{x}$ for a fixed Ω. A larger Ω raises $A^*(\mathbf{x})$.

Two definitions equivalent with the one-to-one mapping, $s(z) \leftrightarrow s^*(y)$, defined by $s^* \equiv s(s^*/y)$ or $s \equiv s^*(s/z)$.

CES if $s^*(y) = \gamma^{1/\sigma} y^{1-1/\sigma}$; CoPaTh if $s^*(y) = \left[(\gamma)^{\frac{\rho-1}{\rho}} + (y\bar{z})^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$ with $\rho \in (0,1)$.

Production Function: $X(\mathbf{x}) = c^* A^*(\mathbf{x}) \exp\left\{\int_\Omega \left[\int_0^{x_\omega/A^*(\mathbf{x})} s^*(\xi)\frac{d\xi}{\xi}\right] d\omega\right\}$

Note: Our 2020 paper proved

$$\left[1 - \frac{d\ln s(z)}{d\ln z}\right]\left[1 - \frac{d\ln s^*(y)}{d\ln y}\right] = 1$$

Our 2017 paper proved that $X(\mathbf{x})$ is quasi-concave & that $A^*(\mathbf{x})/X(\mathbf{x}) = P(\mathbf{p})/A(\mathbf{p}) \neq c$ for any $c > 0$ unless CES

- ✓ $A^*(\mathbf{x})$, the measure of *competitive pressures*, fully captures *cross quantity effects* in the inverse demand system
- ✓ $X(\mathbf{x})$, the measure of output, captures the *output implications* of input changes

Three Parametric Families of H.S.A. (Appendix D)

Generalized Translog For $\eta > 0, \sigma > 1$	$s(z) = \gamma\left(-\frac{\sigma-1}{\eta}\ln\left(\frac{z}{\bar{z}}\right)\right)^{\eta}$; $z < \bar{z} \equiv \beta e^{\frac{\eta}{\sigma-1}}$	$1-\frac{1}{\zeta(z)} = \frac{\eta}{\eta - \ln\left(\frac{z}{\bar{z}}\right)} \Rightarrow \begin{matrix} \mathcal{E}_{\mu}(\cdot) < 0 \\ \mathcal{E}'_{\mu}(\cdot) = \rho'(\cdot) < 0 \end{matrix}$ satisfying **A2;** violating **A3.**

Translog is the special case where $\eta = 1$. CES is the limit case, as $\eta \to \infty$, while holding $\beta > 0$ and $\sigma > 1$ fixed.

Constant Pass-Through (CoPaTh) For $0 < \rho < 1, \sigma > 1$	$s(z) = \gamma\sigma^{\frac{\rho}{1-\rho}}\left[1-\left(\frac{z}{\bar{z}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}$; $\bar{z} \equiv \beta\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\rho}{1-\rho}}$	$1-\frac{1}{\zeta(z)} = \left(\frac{z}{\bar{z}}\right)^{\frac{1-\rho}{\rho}} \Rightarrow \begin{matrix} \mathcal{E}_{\mu}(\cdot) < 0 \\ \mathcal{E}'_{\mu}(\cdot) = \rho'(\cdot) = 0 \end{matrix}$; satisfying **A2** & weak **A3;** violating strong **A3**

CES is the limit case, as $\rho \to 1$, while holding $\beta > 0$ and $\sigma > 1$ fixed.

Power Elasticity of Markup Rate (PEM)/Fréchet Inverse Markup Rate (FIM) For $\kappa \geq 0$ and $\lambda > 0$	$s(z) = \exp\left[\int_{z_0}^{z}\frac{c}{c-\exp\left[-\frac{\kappa\bar{z}^{-\lambda}}{\lambda}\right]\exp\left[\frac{\kappa\xi^{-\lambda}}{\lambda}\right]}\frac{d\xi}{\xi}\right]$	$1-\frac{1}{\zeta(z)} = c\exp\left[\frac{\kappa\bar{z}^{-\lambda}}{\lambda}\right]\exp\left[-\frac{\kappa z^{-\lambda}}{\lambda}\right]$ $\Rightarrow \mathcal{E}_{\mu}(\cdot) < 0; \mathcal{E}'_{\mu}(\cdot) = \rho'(\cdot) > 0$ satisfying **A2** and strong **A3** for $\kappa > 0$ and $\lambda > 0$.

CES for $\kappa = 0$; $\bar{z} = \infty$; $c = 1-\frac{1}{\sigma}$; CoPaTh for $\bar{z} < \infty$; $c = 1$; $\kappa = \frac{1-\rho}{\rho} > 0$, and $\lambda \to 0$.

Generalized Translog:

$$s(z) = \gamma\left(1 - \frac{\sigma-1}{\eta}\ln\left(\frac{z}{\beta}\right)\right)^{\eta} = \gamma\left(-\frac{\sigma-1}{\eta}\ln\left(\frac{z}{\bar{z}}\right)\right)^{\eta};\ z < \bar{z} \equiv \beta e^{\frac{\eta}{\sigma-1}}$$

$$\Longrightarrow \zeta(z) = 1 + \frac{\sigma-1}{1 - \frac{\sigma-1}{\eta}\ln\left(\frac{z}{\beta}\right)} = 1 - \frac{\eta}{\ln\left(\frac{z}{\bar{z}}\right)} > 1$$

$$\Longrightarrow \eta z\zeta'(z) = [\zeta(z)-1]^2 \Longrightarrow \frac{z\zeta'(z)}{[\zeta(z)-1]\zeta(z)} = \frac{1}{\eta}\left[1 - \frac{1}{\zeta(z)}\right] = \frac{1}{\eta - \ln\left(\frac{z}{\bar{z}}\right)}$$

satisfying **A2** but violating **A3.**

- CES is the limit case, as $\eta \to \infty$, while holding $\beta > 0$ and $\sigma > 1$ fixed, so that $\bar{z} \equiv \beta e^{\frac{\eta}{\sigma-1}} \to \infty$.
- Translog is the special case where $\eta = 1$.
- $z = Z\left(\frac{\psi}{A}\right)$ is given as the inverse of $\frac{\eta z}{\eta - \ln(z/\bar{z})} = \frac{\psi}{A}$;
- If $\eta \geq 1$, employment is globally decreasing in z;
- If $\eta < 1$, employment is hump-shaped with the peak, given by $\hat{z}/\bar{z} = \frac{\hat{\psi}}{(1-\eta)\bar{z}A} = \exp\left[-\frac{\eta^2}{1-\eta}\right] < 1$, decreasing in η.

Constant Pass-Through (CoPaTh): Matsuyama-Ushchev (2020b). For $0 < \rho < 1, \sigma > 1, \bar{z} \equiv \beta \left(\frac{\sigma}{\sigma-1}\right)^{\frac{\rho}{1-\rho}}$

$$s(z) = \gamma\sigma^{\frac{\rho}{1-\rho}}\left[1 - \left(\frac{z}{\bar{z}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}} \Rightarrow 1 - \frac{1}{\zeta(z)} = \left(\frac{z}{\bar{z}}\right)^{\frac{1-\rho}{\rho}} < 1 \Rightarrow \mathcal{E}_{1-1/\zeta}(z) = -\mathcal{E}_{\zeta/(\zeta-1)}(z) = \frac{1-\rho}{\rho} > 0$$

satisfying **A2** and the weak form of **A3** (but not the strong form). Then, for $\psi/A < \bar{z}$,

$$p_\psi = (\bar{z}A)^{1-\rho}(\psi)^\rho; \qquad Z\left(\frac{\psi}{A}\right) = (\bar{z})^{1-\rho}\left(\frac{\psi}{A}\right)^\rho;$$

$$\sigma\left(\frac{\psi}{A}\right) = \frac{1}{1-(\psi/\bar{z}A)^{1-\rho}}; \qquad \rho\left(\frac{\psi}{A}\right) = \rho$$

$$r\left(\frac{\psi}{A}\right) = \gamma\sigma^{\frac{\rho}{1-\rho}}\left[1 - \left(\frac{\psi}{\bar{z}A}\right)^{1-\rho}\right]^{\frac{\rho}{1-\rho}}; \ \pi\left(\frac{\psi}{A}\right) = \gamma\sigma^{\frac{\rho}{1-\rho}}\left[1 - \left(\frac{\psi}{\bar{z}A}\right)^{1-\rho}\right]^{\frac{1}{1-\rho}}; \ \ell\left(\frac{\psi}{A}\right) = \gamma\sigma^{\frac{\rho}{1-\rho}}\left(\frac{\psi}{\bar{z}A}\right)^{1-\rho}\left[1 - \left(\frac{\psi}{\bar{z}A}\right)^{1-\rho}\right]^{\frac{\rho}{1-\rho}}$$

with

- a constant pass-through rate, $0 < \rho < 1$.
- Employment hump-shaped with $\hat{z}/\bar{z} = (1-\rho)^{\frac{\rho}{1-\rho}} > \hat{\psi}/\bar{z}A = (1-\rho)^{\frac{1}{1-\rho}}$, both decreasing in ρ.
- CES is the limit case, as $\rho \to 1$, while holding $\beta > 0$ and $\sigma > 1$ fixed, so that $\sigma(\psi/A) \to \sigma$; $\bar{z} \equiv \beta\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\rho}{1-\rho}} \to \infty$.

Power Elasticity of Markup Rate (PEM)/Fréchet Inverse Markup Rate (FIM): For $\kappa \geq 0$ and $\lambda > 0$

$$s(z) = \exp\left[\int_{z_0}^{z} \frac{c}{c - \exp\left[-\frac{\kappa \bar{z}^{-\lambda}}{\lambda}\right]\exp\left[\frac{\kappa \xi^{-\lambda}}{\lambda}\right]} \frac{d\xi}{\xi}\right]$$

with either $\bar{z} = \infty$ and $c \leq 1$ or $\bar{z} < \infty$ and $c = 1$. Then,

$$1 - \frac{1}{\zeta(z)} = c \exp\left[\frac{\kappa \bar{z}^{-\lambda}}{\lambda}\right]\exp\left[-\frac{\kappa z^{-\lambda}}{\lambda}\right] < 1 \Rightarrow \mathcal{E}_{1-1/\zeta}(z) = -\mathcal{E}_{\zeta/(\zeta-1)}(z) = \kappa z^{-\lambda}$$

satisfying **A2** and the strong form of **A3** for $\kappa > 0$ and $\lambda > 0$.

CES for $\kappa = 0$; $\bar{z} = \infty$; $c = 1 - \frac{1}{\sigma}$; CoPaTh for $\bar{z} < \infty$; $c = 1$; $\kappa = \frac{1-\rho}{\rho} > 0$, and $\lambda \to 0$**.**

- $\rho\left(\frac{\psi}{A}\right) = \frac{1}{1+\kappa(z_\psi)^{-\lambda}}$, with $z_\psi = Z\left(\frac{\psi}{A}\right)$ given implicitly by $c \exp\left[\frac{\kappa \bar{z}^{-\lambda}}{\lambda}\right] z_\psi \exp\left[-\frac{\kappa (z_\psi)^{-\lambda}}{\lambda}\right] \equiv \frac{\psi}{A}$,
- $\frac{\partial^2 \ln \rho(\psi/A)}{\partial A \partial \psi} \lesseqgtr 0 \Leftrightarrow (\kappa)^{\frac{1}{\lambda}} \gtreqless z_\psi = Z\left(\frac{\psi}{A}\right) \Leftrightarrow \frac{\psi}{A} \lesseqgtr (\kappa)^{\frac{1}{\lambda}} c \exp\left[\frac{\kappa \bar{z}^{-\lambda} - 1}{\lambda}\right]$; Log-sub(super)modular among more (less) efficient firms. In particular, if $\bar{z} < (\kappa)^{\frac{1}{\lambda}}$, $\frac{\partial^2 \ln \rho(\psi/A)}{\partial A \partial \psi} < 0$ for all $\psi/A < Z(\psi/A) < \bar{z} < \infty$.
- Employment hump-shaped with the peak at $\hat{z} = Z\left(\frac{\hat{\psi}}{A}\right) < \bar{z}$, given implicitly by

$$c\left(1 + \frac{\hat{z}^{\lambda}}{\kappa}\right)\exp\left[-\frac{\kappa \hat{z}^{-\lambda}}{\lambda}\right]\exp\left[\frac{\kappa \bar{z}^{-\lambda}}{\lambda}\right] = 1 \Leftrightarrow \left(1 + \frac{\hat{z}^{\lambda}}{\kappa}\right)\hat{z} = \frac{\hat{\psi}}{A}.$$